

大葉大學 96 學年度 研究所碩士班甄試 招生考試試題紙

系 所 別	組 別	考 試 科 目 (中文名稱)	考 試 日 期	節 次	備 註
電 機 工 程		工 程 數 學	12 月 23 日	第 一 節 8:30~10:00	莫乙夏

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

1. Please solve the following first-order differential equations:

(a) $(x^2 + y^2 + x)dx + (xy)dy = 0$ (10%)

(b) $(1 + x^2)(dy - dx) = (2xy)dx$ (10%)

2. A second-order differential equation with boundary conditions is given as follows:

$x^2 y'' - xy' + y = x \ln x$ (10%)

B.C. $y(1) = 0$ and $y'(1) = 0$

If the solution with a form likes $y(x) = (c_1 + c_2 \ln x)x + c_3 x \ln^3 x$, please find the constants c_1, c_2 and c_3 ?

3. Laplace transform:

(a) Use **convolution theorem** to find the inverse of the following Laplace's equation:

$F(s) = \frac{1}{(s^2 + 1)^2}$ (10%)

(b) Use Laplace transform to solve the following integral equation:

$y'(t) + 5 \int_0^t \cos 2(t - \tau)y(\tau)d\tau = 10 \quad y(0) = 2$ (10%)

4. Sturm-Liouville boundary value problem and expansion of Fourier series:

(a) Solve the following Sturm-Liouville boundary value problem (SL-BVP): (15%)

$y'' + \lambda y = 0 \quad y(x) = y(x + 2\pi)$ and $-\pi \leq x \leq \pi$

B.C. $y(-\pi) = y(\pi)$

$y'(-\pi) = y'(\pi)$

Please discuss the following conditions in detail, i.e., $\lambda > 0$, $\lambda = 0$ and $\lambda < 0$, respectively.

(b) Please expand the following function $f(x)$ in a Fourier series: (15%)

$f(x) = \begin{cases} -1 & \text{if } -\pi \leq x < 0 \\ +1 & \text{if } 0 \leq x \leq \pi \end{cases}, f(x) = f(x + 2\pi)$

5. Complex integral and Laurent series

(a) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series valid for $0 < |z+1| < 2$ (10%)

(b) Find the complex integration $\frac{1}{2\pi i} \oint_C \frac{dz}{(z^{50} + 1)(z - 3)}$ and $C: |z| = \infty$ (10%)