大葉大學 95 學年度 研究所碩士班 招生考試試題紙

系所別	組別	考試科目（中文名稱）	考試日期	節次	備註
資訊工程	甲	離散數學	4月23日	第二節	共乙頁

10:30 ~ 12:00

註：不可使用計算機，作答需**詳列過程及解釋原因**，否則僅部分給分，太複雜的算式不必乘開。

1. Given a set S, the <u>*power set*</u> of S is the set of all subsets of the set S. The power set of S is denoted by $P(S)$.
   (a) (5%) What is the power set of the set $\{a, b, c\}$?
   (b) (10%) Let A be a set of n elements, $n \in \mathbf{Z}^+$. How many elements does $P(A)$ have? Why?

2. (15%) Describe an algorithm to input an $m \times n$ matrix $A = [a_{ij}]$ and an $n \times k$ matrix $B = [b_{ij}]$, where $m, n, k \in \mathbf{Z}^+$, and compute the product $C = A \times B$.

3. (a) (7%) Determine the coefficient of xyz^2 in $(x + y + z)^4$.
   (b) (8%) What is the coefficient of xyz^{m-2} in $(x + y + z)^m$ for any positive integer $m > 4$?

4. (10%) Let n be a positive integer. Prove that

$$\binom{n}{0} + 2\binom{n}{1} + 2^2\binom{n}{2} + 2^3\binom{n}{3} + \cdots + 2^n\binom{n}{n} = 3^n.$$

5. (10%) Solve the recurrence relation $a_n = 5a_{n-1} + 6a_{n-2}$, where $n \ge 2$, $a_0 = 1$, $a_1 = 3$.

6. For $A = \{1, 2, 3, 4, 5, 6\}$, $\mathcal{R} = \{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4), (4, 5), (5, 4), (5, 5), (6, 6)\}$ is a relation defined on A.
   (a) (6%) Show that $\mathcal{R}$ is an equivalence relation.
   (b) (4%) What are the equivalence classes [2] and [4] under this equivalence relation?

7. (10%) Show that any subset of size 6 from the set $S = \{1, 2, 3, \ldots, 9\}$ must contain two elements whose sum is 10.

8. (a) (10%) Determine whether or not the following graphs are isomorphic.
   (b) (5%) Is G a planar graph?