

大葉大學 95 學年度 研究所碩士班甄試

招生考試試題紙

| 系所別 | 組別 | 考試科目 (中文名稱) | 考試日期 | 節次 | 備註 |
|------|----|----------------|--------|-----|-----|
| 電機工程 | | 工程數學 | 12月19日 | 第一節 | 共2頁 |

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

1. (10%) Find the general solution of the first order differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 \cos 3x.$$

2. (10%) The solution of the second order differential equation $y'' - ay' + by = 0$ with initial condition $y(0) = 1$ and $y'(0) = 3$ is $y(t) = Ae^{-t} \cos 2t + Be^{-t} \sin 2t$ where a, b, A, B are constants. Find the values of a, b, A, B

3. (12%) Solve the integral-differential equation by the method of Laplace transform

$$y'(x) = 1 - \int_0^x y(t)e^{-2(x-t)} dt; y(0) = 1$$

4. (18%) Let $H(\omega)$, $F(\omega)$, $G(\omega)$ be the **Fourier Transform** of $h(t)$, $f(t)$ and $g(t)$ respectively

- (6%) (1) If $g(t)$ is related to $f(t)$ as follows:

$$g(t) = 2f(t) + \frac{1}{2}[f(t-2) + f(t+2)]$$

What is the relationship between $F(\omega)$ and $G(\omega)$.

- (12%) (2) If $g(t)$ be the convolution of $f(t)$ and $h(t)$. If $f(t) = e^{i\omega_1 t} + e^{-i\omega_2 t}$, express $g(t)$ in terms of ω_1, ω_2 , and $H(\omega)$.

5. (12%) Using (a) direct calculation (b) Green's theorem in the plane evaluate

$$\oint_C [(3x^2 + y)dx + 4y^2 dy]$$

C is the boundary of the triangle with vertices (0,0), (1,0), (0,2) in counterclockwise

6. (10%) $f(x) = 2x, g(x) = 3 + cx, 0 \leq x \leq 1$

(1) What is the value of c so that $f(x)$ and $g(x)$ are orthogonal?

(2) What are the normalized functions of $f(x)$ and $g(x)$ respectively?

7. (10%) Solve the eigenvalue problem

$$y'' + \lambda y = 0, y(0) = y(1) = 0$$

8. (8%) Find the determinant of the $n \times n$ matrix **B** that has p 's on the diagonal and q 's elsewhere:

$$\mathbf{B} = \begin{bmatrix} p & q & \cdots & q \\ q & p & \cdots & q \\ \vdots & \vdots & \ddots & \vdots \\ q & q & \cdots & p \end{bmatrix}$$

9. (10%) Let λ and μ be two distinct eigenvalues of a real square matrix **A**, and let \mathbf{x} and \mathbf{y} be corresponding eigenvectors of **A**, Show that if in addition, **A** is symmetric, then \mathbf{x} and \mathbf{y} are orthogonal