大葉大學 95 學年度 研究所碩士班甄試 招生考試試題紙					
系 所 別	組 別	考 試 科 目 （中 文 名 稱）	考 試 日 期	節 次	備 註
機械工程研究所 機電自動化研究所 車輛工程研究所	甲乙丙 甲 甲	工程數學	12 月 19 日	第 一 節	共一頁

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

1. Find the general solution of the differential equation (15%)

   $y'' + y = t^2$ (which $y' = \frac{dy}{dt}$)

2. Using the Laplace transform, solve the following problem (20%)

   $y'' + 3y' + 2y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 0$ (which $y' = \frac{dy}{dt}$)

   which $\delta(t-1)$ is Dirac's delta function, by definition $\delta(t-1) = \infty$ for $t = 1$,
   $\delta(t-1) = 0$ for $t \neq 1$

3. Please find eigenvalues and corresponding eigenvectors of the following matrix **A** (15%)

   $$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

4. Find the Fourier series of the function (15%)

   $f(x) = x$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$

5. Scale function $f(x, y, z) = x^2 y - yz^2$, vector function $\mathbf{v}(x, y, z) = y\mathbf{i} + z\mathbf{j} + x\mathbf{k}$, find

   (a) div (grad f) (5%)
   (b) div ($\mathbf{v}\, f$) (5%)
   (c) div (curl $\mathbf{v}$) (5%)

6. Solve the following partial differential equations (PDE) by separation of variables (20%)

   $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad 0 \le x \le L, \quad t > 0$$

   Boundary conditions: $u(0, t) = 0, \quad u(L, t) = 0$
   Initial conditions: $u(x, 0) = f(x)$