

大葉大學 九十四 學年度 研究所碩士班 招生考試試題紙

系 所 別	組 別	考 試 科 目 (中 文 名 稱)	考 試 日 期	節 次	備 註
資訊管理系	乙	離散數學	3月27日	第二節 10:30~12:00	共1頁

註：考生不可攜帶計算機或其他資料作答，並且答題應詳列計算步驟，否則一概不予計分。

- (a) Show that the minimal number of moves needed to solve the Tower of Hanoi puzzle is $2^n - 1$, if there are n disks and 3 pegs. (5%)

(b) Find the smallest number of moves needed to solve the Tower of Hanoi puzzle with n disks and n pegs. (10%)
- Use the difference table to find the next term of the following sequence.
-1, -2, 3, 20, 55, 114, 203 (5%)
- How many pairs of rabbits can be bred from one pair of rabbits in a year if each pair of rabbits breeds another pair each month, and begins breeding in the second month after birth? (10%)
- Use the Principle of Mathematical Induction to prove that
 $n! \geq 2^n$ for $n \geq 4$ (10%)
- Prove the following statement using the contrapositive :
If m is an integer and m^2 is divisible by 5, then m is divisible by 5. (10%)
- Which of the following relations on the set $A = \{1, 2, 3, 4, 5\}$ are antisymmetric? Which are partial orderings? Why? (10%)

(a) $R_1 = \{(1, 2), (3, 4)\}$

(b) $R_2 = A \times A$

(c) $R_3 = \{(1, 2), (3, 4), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)\}$

(d) $R_4 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 5), (5, 1)\}$
- Derive a recurrence relation for the following a_n .

(a) Let a_n denote the number of binary tree with n nodes. (5%)

(b) Let a_n denote the number of nodes of complete binary tree with height n . (5%)
- (a) Solve the recurrence relation $a_{n+1}^2 = 5a_n^2$, $a_n \geq 0$, $a_0 = 2$. (7%)

(b) Compute a_{12} by the formula derived from (a). (3%)
- Solve the recurrence relation $F_{n+2} = F_{n+1} + F_n$, where $n \geq 0$ and $F_0 = 0$, $F_1 = 1$. (10%)
- Let $f: Z \rightarrow Z$ be defined by $f(n) = n^3 - 4n$. Is f a one-to-one correspondence? Prove your answer. (10%)