註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

註2：作答需**詳列過程**及**解釋原因**，否則不予計分，太複雜的算式不必乘開。

1. (20%) Let $A = \{a, b, c, d\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$.

    (a) (5%) How many functions are there from A to B?

    (b) (5%) How many <u>one-to-one</u> functions are there from A to B?

    (c) (10%) How many <u>onto</u> functions $f: B \to A$ satisfying $f(1) = a$?

2. (10%) Let n be a positive integer and define a relation R on the set S of all nonnegative integers by aRb if and only if a and b have the same remainder when divided by n.

    (a) (6%) Prove that R is an equivalence relation on S.

    (b) (4%) How many different equivalence classes does R give?

3. (15%) Solve the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ with initial conditions $a_0 = -5$, $a_1 = 4$ and $a_2 = 88$.

4. (15%) Let $\begin{Bmatrix} n \\ k \end{Bmatrix}$ denote the number of ways to partition the set $\{1, 2, \ldots, n\}$ into k non-empty sets.

    For example, $\begin{Bmatrix} 4 \\ 2 \end{Bmatrix} = 7$ since there are 7 ways to partition the set $\{1, 2, 3, 4\}$ into 2 non-empty sets:

    $\{1\}, \{2,3,4\}$; $\{2\}, \{1,3,4\}$; $\{3\}, \{1,2,4\}$; $\{4\}, \{1,2,3\}$; $\{1,2\}, \{3,4\}$; $\{1,3\}, \{2,4\}$; $\{1,4\}, \{2,3\}$.

    Show that $\begin{Bmatrix} n \\ k \end{Bmatrix} = \begin{Bmatrix} n-1 \\ k-1 \end{Bmatrix} + k\begin{Bmatrix} n-1 \\ k \end{Bmatrix}$.

5. (15%) An <u>edge coloring</u> of a graph is an assignment of colors to edges so that edges incident with a common vertex are assigned different colors. The <u>edge chromatic number</u> of a graph is the smallest number of colors that can be used in an edge coloring of the graph. Find the edge chromatic numbers of C_n and $K_{n,n}$.

6. (10%) Let $n_1, n_2, \ldots, n_r$ be positive integers. Show that if $n_1 + n_2 + \ldots + n_r - r + 1$ objects are placed into r boxes, then for some i, $i = 1, 2, \ldots, r$, the ith box contains at least n_i objects.

7. (15%) A <u>mode</u> of a list of integers is an element that occurs at least as often as each of the other elements. Describe an algorithm that finds a mode in a list of integers.