

# 大葉大學九十四學年度碩士班甄試試題紙

所 別	組別	考 試 科 目 (中 文 名 稱)	考 試 日 期	考 試 時 間	備 註
環境工程研究所		微積分	12月13日	09:00~10:30	共乙頁

註：備註欄若未註明可攜帶計算機或其他輔助工具作答時，考生一律不准攜帶。

1. (21%) Find the derivatives of the following functions:

(a).  $y = x^{\sqrt{x}}$

(b).  $y = \int_0^{\sin x} \frac{dt}{\sqrt{1-t^2}}$ , and  $|x| < \frac{\pi}{2}$

(c).  $y^2 - x^2 - \sin xy = 0$

2. (40%) Evaluate the following integrals:

(a).  $\int \frac{6x+7}{(x+2)^2} dx$

(b).  $\int e^x \cos x dx$

(c).  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$

(d).  $\iint_R e^{x^2+y^2} dy dx$ , where R is the semicircular region bounded by the x-axis and the curve  $y = \sqrt{1-x^2}$ . (hint: use polar coordinates)

3. (10%)  $y = |x|$ , find the average value of the function over the interval  $[-1, 1]$ .

4. (10%)

(a). Use the Integral Test to prove the convergence of  $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

(b). Use the Ratio Test to prove the divergence of  $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

5. (10%) The decay of an individual hazardous constituent in the environment can be described as a first-order function as follows:

$$\frac{dC}{dt} = -kC, \text{ where } C = \text{concentration at time } t$$

$t = \text{time}$

$k = \text{first-order reaction rate constant}$

Prove the half-life, at which time the concentration drops to half its original concentration, of an individual constituent is equal to  $\frac{0.69}{k}$ .

6. (9%) Set up a triple integral for the volume of the upper hemisphere

given by  $z = (1-x^2-y^2)^{\frac{1}{2}}$ .