大葉大學 九十三 學年度 研究所碩士班 招生考試試題紙					
系 所 別	組 別	考 試 科 目 (中文名稱)	考試 日期	節 次	備 註
資訊管理系	乙	離散數學	3月28日	第 2 節	共二頁 P.2-1

10:30 ~ 12:00

註：考生不可攜帶計算機或其他資料作答，並且答題應詳列計算步驟，否則一概不於計分。

1. Suppose you wish to prove that the following is true for all positive integers n by using the Principle of Mathematical Induction:
   $1 * 1! + 2 * 2! + 3 * 3! + \ldots + n * n! = (n + 1)! - 1$
   (a) Use $P(7)$ to prove $P(8)$. (3%)
   (b) Use the Principle of Mathematical Induction to prove that $P(n)$ is true for all positive integers n. (7%)

2. Twenty-four people in a town have decided to form three clubs. Some are more social than others, and the clubs are chosen so that half of the people belong to each club, one-third of the people to each pair of clubs, and one-fourth of the people to all three clubs. How many people belong to at least one of the clubs? How many people belong to two or more clubs? (10%)

3. (a) Use the Euclidean Algorithm to compute the greatest common divisor and the least common multiple of the pair of integers $(m, n) = (130, 23)$, and to write the greatest common divisor in the form $g.c.d.(m, n) = Am + Bn$. (10%)
   (b) By using the results of (a), find the following solutions of x:
   $130x = 1 \pmod{23}$ (5%)

4. What is the Chinese Remainder Theorem (CRT) ? By using the CRT to solve the following positive integer x :
   $x = 3 \bmod 5 = 2 \bmod 7 = 7 \bmod 11$ (10%)

5. Prove that $\sqrt{6}$ is irrational using the contradiction. (10%)

6. Prove that Fibonacci numbers can be expressed by the formula
   $$F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$$ (10%)

7. The functionality of a gate can be defined by its outout value for every possible combination of input values. Please answer the following questions.
   (a) How can we use 2 NAND gates to implement one AND gate? (3%)
   (b) How can we use 3 NAND gates to implement a circuit which evaluates
   E = (A AND B) OR (C AND D) ? (7%)

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8. Find a minimal sum of products expression equivalent to the expression

$$pq + pr + q'r \qquad (10\%)$$

9. Solve the recurrence relation

$$\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}} \quad , a_0 = 1, a_1 = 1 \qquad (10\%)$$

10. Let A, B, and C be 3 arbitrary sets. Show that

$(A - B) - C = A - (B \cup C)$ (5%)