<table>
<tr><td colspan="6">大葉大學九十三學年度碩士班甄試試題紙</td></tr>
<tr><td>所　別</td><td>組別</td><td>考試科目
（中文名稱）</td><td>考試日期</td><td>考試時間</td><td>備註</td></tr>
<tr><td>機械工程研究所
機電自動化研究所
車輛工程研究所</td><td>甲、乙、丙
甲
甲</td><td>工程數學</td><td>12月8日</td><td>9:00 ~ 10:30</td><td>不准攜帶計算機或輔助工具
（試題共一頁）</td></tr>
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注意：(1) 答題應詳列運算步驟，否則不予計分。

(2) 答案卷務必標明題號，無題號之計算視為草稿，不予計分。

1. (10 分) Obtain a general solution of the linear differential equation $y' + 2y = 3xe^{x}$.

2. (20 分) Solve the initial value problem： $(x^2D^2 - 3xD + 3)y = 3\ln x - 1$; $y(1) = 0$, $y'(1) = 1$, where $D = \dfrac{d}{dx}$.

3. (15 分) Use the Laplace transform to solve the initial value problem: $y'' + 4y = f(t)$; $y(0) = 1$, $y'(0) = 0$, where $f(t) = \begin{cases} 0 & 0 \le t < 4 \\ 3 & t \ge 4 \end{cases}$

[Hint: First, express the function $f(t)$ in terms of the unit step function (or called Heaviside function).]

4. (15 分) (a) Write the mathematical formula for the divergence theorem of Gauss.

(b) Apply the divergence theorem to $\vec{F} \times \vec{C}$ to show that $\iint_A \vec{n} \times \vec{F}\, dA = \iiint_V \nabla \times \vec{F}\, dV$, where $\vec{C}$ is an arbitrary constant vector, $\vec{F}(x,y,z)$ is a differentiable vector function, and $\vec{n}$ is the outer unit normal vector of A.

5. (15 分) For the matrix $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -3 & -1 \end{bmatrix}$, (a) find its eigenvalues λ_1 and $\lambda_2\ (>\lambda_1)$, (b) find the corresponding eigenvectors, (c) find a matrix $\mathbf{S}$ and its inverse $\mathbf{S}^{-1}$ so that $\mathbf{S}^{-1}\mathbf{AS} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$.

6. (10 分) Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$ and the Fourier integral representation of $f(x)$ be given as $f(x) = \int_0^\infty [A(\omega)\cos\omega x + B(\omega)\sin\omega x]d\omega$. Find the coefficient $A(\omega)$.

7. (15 分) Use the residue theorem to evaluate $\dfrac{1}{2\pi i}\oint_C \dfrac{e^{zt}}{z^2(z^2 + 2z + 2)}dz$, where C is the closed curve $|z| = 3$ taken counterclockwise direction, and $i = \sqrt{-1}$. Express the result in terms of t, e^{-t}, and $\cos(t)$.