大葉大學九十三學年度轉學招生考試試題紙

系　　　組　　　別	日＼第二部	年級	考　試　科　目（中文名稱）	考試日期	節次	備註
機械、工工、電機 生科、環工、資工	日、第二部 日	二	微　積　分	7月19日	3	共乙頁

註：考生可否攜帶計算機或其他資料作答，請在備註欄註明（如未註明，一律不准攜帶）

11:10 ~ 12:30

1. (20%) Determine the limits

(a) $\lim\limits_{h \to 0} \dfrac{\sqrt{2+h}-\sqrt{2}}{h}$

(b) $\lim\limits_{x \to \infty} \dfrac{1}{x \ln x} \displaystyle\int_{1}^{x} \ln t\, dt$

2. (20%) Find the derivative $\dfrac{dy}{dx}$

(a) $y = -4\sqrt{x} + \dfrac{1}{\sqrt{x}} + 7 + \dfrac{\cos x}{1+\sin x}$

(b) $y^2 - x^2 - \sin(xy) = 0$

3. (20%) Evaluate the integrals

(a) $\displaystyle\int_{\pi/4}^{\pi/2}\left(-4\sqrt{x} + \dfrac{1}{\sqrt{x}} + 7 + \dfrac{\cos x}{1+\sin x}\right)dx$

(b) $\displaystyle\int_{1}^{\infty} \dfrac{\ln x}{x^2}dx$

4. (10%) Find the absolute maximum and minimum values of the function
$$f(x,y) = 2 + 2x + 2y - x^2 - y^2$$
on the triangular plate in the first quadrant bounded by the lines
$$x = 0, \quad y = 0, \quad and \quad y = 9 - x.$$

5. (10%) Evaluate the integral $\displaystyle\int_{-2}^{2}\int_{-\sqrt{(4-x^2)/2}}^{\sqrt{(4-x^2)/2}}\int_{x^2+3y^2}^{8-x^2-y^2} dz\,dy\,dx$.

6. (10%) Find the Maclaurin Series for $\cos 2x$.
(Hint: you may use the formula
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots$$
$$= \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} \quad , \text{ for all real } x.)$$

7. (10%)
(a) Find the intervals of convergence of the series
$$y = 1 + \frac{1}{6}x^3 + \frac{4}{720}x^6 + \cdots + \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{(3n)!}x^{3n} + \cdots,$$

(b) Show that the function defined by the series satisfies a differential equation of the form $\dfrac{d^2 y}{dx^2} = x^a y + b$
and find the values of the constants a and b.