

# 大葉大學九十二學年度碩士班甄試試題紙

所 別	組別	考 試 科 目 (中 文 名 稱)	考 試 日 期	考 試 時 間	備 註
環境工程學系 碩士班	甲	微積分	12月9日	9:00~10:30	共乙頁

註：備註欄若未註明可攜帶計算機或其他資料作答時，考生一律不准攜帶。

1. Find the points at which  $f(x)$  is discontinuous. Sketch the graph of  $f(x)$ . (10 points)

(a) 
$$f(x) = \begin{cases} 2x+1 & \text{if } x \leq -1 \\ 3x & \text{if } -1 < x \leq 1 \\ 2x-1 & \text{if } 1 \leq x \end{cases}$$

(b) 
$$f(x) = \begin{cases} (x-1)^3 & \text{if } x < 0 \\ (x+1)^3 & \text{if } x \geq 0 \end{cases}$$

2. The function  $g(x)$  is a twice differentiable function, find  $f(x)''$  (second derivative of  $f(x)$ )

in terms of  $g$ ,  $g'$  (first derivative of  $g(x)$ ) and  $g''$ . (10 points)

(a)  $f(x) = xg(x)$       (b)  $f(x) = g(x)/x$

3. Differentiate the function. (10 points)

(a)  $f(x) = x^4 - 6x^2 + 7$       (b)  $g(x) = \frac{6}{x^{\frac{1}{3}}}$

4. Find the first and second derivatives of the function. (10 points)

(a)  $f(x) = x^8 - 6x^6 + 7x^5$       (b)  $g(x) = \frac{6}{\sqrt{1-x}}$

5. Evaluate the following integrals. (10 points)

(a)  $\int \frac{dx}{x^2 - 9}$       (b)  $\int_{-\infty}^0 xe^x dx$

6. Evaluate the following limits. (10 points)

(a)  $\lim_{x \rightarrow \infty} \frac{2e^x}{x^2}$       (b)  $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$

7. Find the limit if it exists, or show that the limit does not exist. (5 points)

$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2 + y^2}$

8. If  $f(x,y) = x^3 - x^2y^2 + 3y^3$ , find  $f_{xx}(3,1)$ ,  $f_{yx}(3,1)$ . (5 points)

9. If  $u = x^4y + 3y^2z^3$ ,  $x = rse^t$ ,  $y = rs^2e^{-t}$ ,  $z = r^2s \sin t$ , find the value of  $\frac{\partial u}{\partial s}$  when  $r = 2$ ,  $s = 1$ ,  $t = 0$ . (5 points)

10. If  $z = f(x,y)$  has continuous second-order partial derivatives and  $x = r^2 + s^3$  and  $y = 4rs$ , find  $\frac{\partial^2 z}{\partial r^2}$  (5 points)

11. Find the directional derivative of the function  $f(x,y) = x/y$  at the point  $(6, -2)$  in the direction of the vector  $\mathbf{v} = \langle -1, 3 \rangle$ . (10 points)

12. Evaluate  $\iint_D (2x + 3y) dA$ ,  $D$  is the region bounded by  $y = 2x^2$  and  $y = 1 + x^2$ . (10 points)