資訊工程　　日　　三　　計算數學　　四　　P2-1

1. 請問滿足 $x_1 > 0, x_2 \geq 1, x_3 > 2$ 且 $x_1 + x_2 + x_3 \leq 10$ 的整數解有多少組?(10%)

2. 請問 120! 最後面會有連續幾個 0 ?(10%)

3. 請證明 $\sqrt[3]{3}$ 是無理數 (irrational)。(10%)

4. (a) 請找出 Figure 1 的 minimum spanning tree。(5%)

   (b) 請找出 Figure 1 的一個 spanning tree, 在這個 spanning tree 中由點 a 到其他每個點的距離與原圖相同。(5%)

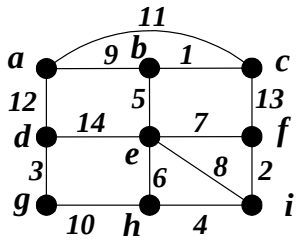

Figure 1:

5. 請寫出 Figure 2 中之 ordered rooted tree 的 preorder traversal, inorder traversal 與 postorder traversal。(10%)

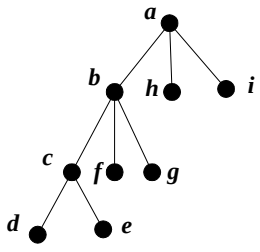

Figure 2:

6. Determine the number of multiplications needed to compute the products $(AB)C$ and $A(BC)$ when matrices A, B, and C are the following sizes: (10%)

   (a) A 2×4, B 4×3, C 3×1

   (b) A 7×97, B 97×2, C 2×3

7. Which of the following collections of vectors are linearly independent in R^3 ? (10%)

   (a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$ (b) $\left\{ \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} \right\}$

8. Determine if the given matrix is invertible. If so, find its inverse. (10%)

   (a). $\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ (b). $\begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

9. If $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 4$, find $\begin{vmatrix} a_1 & a_2 & 8a_3 - 2a_2 \\ b_1 & b_2 & 8b_3 - 2b_2 \\ \frac{1}{2}c_1 & \frac{1}{2}c_2 & 4c_3 - c_2 \end{vmatrix} =?$ (10%)

10. Find the eigenvalues of

    (a). $\begin{bmatrix} 5 & 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 & 1 \\ 0 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (5%) (b). $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (5%)