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# Branch Correlation of *M*-ary Non-Coherent Modulation Schemes in Correlated Statistics Environments

JOY IONG-ZONG CHEN and TAI-WEN HSIEH
*Department of Communication Engineering, Da-Yeh University*
*112 Shan-Jiau Rd., Da-Tsuen, Changhua, Taiwan*

## ABSTRACT

In this study, branch correlation coefficient characterized by correlated Nakagami-*m* distribution is investigated for non-coherent *M*-ary FSK (frequency shift keying). Some new equations, including one for the parabolic cylinder function, are derived. The transmission channels for a wireless radio system are assumed to be frequency non-selective fading channels. Numerical analysis methods are applied to validate the effect of branch correlations with application of the new equations. The results confirm that the branch correlation parameter is not negligible when designing communication systems.

***Key Words*: MFSK, correlation coefficient, correlated Nakagami-*m* channel**

# 研究具相關性統計環境中 *M*-ary 非同調調變方式的分支相關特性

陳雍宗　謝岱彣
大葉大學電信工程研究所
彰化縣大村鄉山腳路 112 號

## 摘　要

本文旨在研究非同調（non-coherent）M-ary 頻率鍵移（frequency shift keying, FSK）調變系統的分支相關係數，其中相關環境假設係處於由相關 Nakami-*m* 統計分布所定性的分支相關環境中。本文推導了一些包含 parabolic cylinder 函數的新方程式，至於傳輸通道是考慮爲非選頻性（frequency non-selective）的衰落通道。本研究中並透過數值分析的方法以確認新方程式中的分支相關效應。

**關鍵詞**：頻率鍵移，相關係數，相關 Nakagami-*m* 通道

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## I. INTRODUCTION

The requirement of high data rates, wide bandwidth and much more capacity becomes the basic conditions for the fast developing wireless communication systems. It is known that the 3G ($3^{rd}$ generation) wireless radio systems have landed the market and it is with the multiple access schemes, which apply the spread spectrum techniques, including the CDMA (code-division multiple-access) and the so-called fast FH (frequency hopping). Especially, due to the much fast speed of transmission rate of the spread spectrum technique, it is the most important one adopted in the area of military. On the other hand, we know the fact that the noncoherent modulation schemes applied in the wireless communication environments are the NFSK (noncoherent frequency shift keying, NFSK) scheme. For the purpose of requirement for high transmission data rate, the NFSK and DPSK are the most two important modulation techniques for multiple-access systems. It is well known that the correlation characteristic happens between the branches in correlated-fading channels. The main aim of the paper focuses on the branch correlation for M-ary noncoherent FSK and DPSK modulation schemes. In this paper, the impact of correlation on the performance of M-ary noncoherent modulator, MFSK, in the correlated Nakagami fading channel is evaluated. For a long time, many of the important studies have employed the Rayleigh distribution to characterize the envelope of fading signals. However, Nakagami-*m* distribution has been thoroughly investigated, since it has been verified as a more versatile model for a variety of fading environments such as urban and suburban radio multipath channels for wireless communication systems [7]. Recently, Lombardo et al. [6] derived an exact expression for the performance of BPSK and NCFSK with prediction MRC (maximal ratio combining) in correlated Nakagami channels. Zhang [11] derived the exact BER expression for BPSK and BFSK systems with MRC over correlated Nakagami-*m* channels. Proakis [9] has studied in MFSK modulation scheme works in the environments with the Rayleigh fading channels. In [10], the authors calculated the BER performance with Chi-square parameters. The performance comparison with encoded (coded) and uncoded for M-ary FSK working in Nakagami-*m* fading statistic was investigated by crepeau [3]. In [2], where the authors analyzed the performance of multiple-cell DS-CDMA systems over correlated Nakagami-*m* fading environments.

In this paper, The BER performance for both encoded and uncoded coding techniques with MFSK modulation operating in Nakagami-*m* fading channel are studied. This paper is organized as follows. In Section II, the system models of uncoded noncoherent modulation are presented. The system performance is analyzed in Section III, in which the BER performance for MFSK is determined for uncoded and coded cases, respectively. The numerical results are illustrated in Section IV. A briefly conclusion is drawn in Section V.

## II. SYSTEM MODEL OF UNCODED NONCOHERENT MODULATION

In wireless communication systems, both Rayleigh and Rician fading models are widely studied, in particular, in urban environments. In this section, the channel model of noncoherent MFSK modulation system is characterized by the correlated Nakagami-*m* statistics distribution, and assumes that the received signal at the receiver output in the interval $[0,T]$ can be expressed as

$$r(t) = R\sqrt{2E_s/T}\cos(\omega_i t + \theta) + n(t) \qquad (1)$$

where $E_S$ is the normalized average received symbol energy, $n(t)$ is AWGN with one-sided power spectral density $N_0$, $\theta$ is uniformly distributed in $[0,2\pi]$, $\omega_i$ denotes one of the $M$ orthogonally spaced frequencies, and $R$ expresses a random variable with Nakagami-*m* distribution whose pdf (probability density function) is given by [7]

$$p(R) = \frac{2m^m R^{2m-1}}{\Gamma(m)\Omega^m} e^{-\left(\frac{m}{\Omega}R^2\right)}, \quad R \ge 0, \quad m \ge \frac{1}{2} \qquad (2)$$

where $\Omega=E[R^2]$, which represents the average power of the signal, $E[\cdot]$ is the expectation operator, and $m=E^2[R^2]/\mathrm{var}[R^2]$ is defined as the ratio of moments, which is called the fading figure. By setting $m=1$, the equation (2) can be reduced to a Rayleigh statistic. For values of $m$ in the range $\frac{1}{2} \le m \le 1$, the pdf has larger percentage tail than a Rayleigh-distribution. If the values of fading figure is greater than one, $m > 1$, the tail of the pdf reduces faster than that of the Rayleigh distribution environments. Assuming that the branches of transmission channel are perfect estimated for MRC diversity is necessary. The pdf of the SNR at the output of an MRC receiver with Rayleigh distributed channel estimation errors has been derived by Gans [5], and is written as

$$p(R) = \frac{(1-\rho^2)^{L-1}}{\Omega} e^{-R/\Omega} \sum_{l=0}^{L-1} \frac{\binom{L-1}{l}}{l!} \left(\frac{R\rho^2}{\Omega(1-\rho^2)}\right)^l \qquad (3)$$

where $\rho$ is called as correlation coefficient, and $\rho\in[0,1]$, is a parameter used for estimation the value between each branch, and $L$ denotes the number of branches. The equation (3) can be applied in the calculation of system performance for the M-ary modulation operating in the correlated fading channels.

In fact, correlation coefficient, $\rho$, represents the channel characteristic used to estimate the affect of the Doppler shift. It represents the perfect branch combination for MRC diversity when $\rho=1$. As the values of $\rho$ decreases gradually, and the analyzed system performance will also be reduced. By substituting $\rho=0$ in the extreme conditions, decline estimation and its actual value are totally independent, and the performance is going not superior as the one that considers the parameters of correlation coefficient between branches for the same system.

## III. SYSTEM PERFORMANCE ANALYSIS

The BER performance is calculated for uncoded and coded coding, and an example for the coded case with Golary coded scheme is presented in this section.

### 1. The Uncoded Case

The wireless communication system works in the slowly non-selective fading channel is supposed in this study. However, no loss of the generality, the normalized value of the average power is adopted, that is, $\Omega=1$, and the energy received at the output of the receiver can be written as $R^2E_S$, where $E_s$ denotes the symbol energy. For the reason mentioned above, the pdf shown in (3) can be reduced and becomes as

$$P(R)=(1-\rho^2)^{L-1}e^{-R}\sum_{l=0}^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{R\rho^2}{(1-\rho^2)}\right)^l \qquad (4)$$

By means of the same approaches adopted in [1], the average SEP (symbol error probability) can be determined by averaging over the conditional symbol error probability under the assumption of correlated Nakagani-*m* fading distributed. Hence, the SEP of the general case for MFSK schemes can be obtained as

$$P_S=\int_0^\infty\sum_{i=1}^{M-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}e^{\frac{-1}{i+1}\frac{E_S R^2}{N_0}}p(R)dR \qquad (5)$$

where $p(R)$ is shown in (4). In order to calculate the BER performance for MFSK, the bit energy can be expressed as $E_b=E_S/\log_2 M$, the BER can be obtained from (5), and can be expressed as

$$P_b=P_s\frac{M}{2(M-1)} \qquad (6)$$

By substituting (3) into (5) and (6), after some algebra operations, the BER for FSK over correlated-Nakagani-*m* statistics can be obtained as (see the Appendix)

$$P_b=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}(1-\rho^2)^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{1-\rho^2}\right)^l$$

$$\times(2\beta)^{\frac{-(l+1)}{2}}\Gamma(l+1)e^{\left(\frac{1}{8\beta}\right)}D_{-v}\left(\frac{1}{\sqrt{2\beta}}\right) \qquad (7)$$

where $\beta=\frac{i}{i+1}\frac{E_b(\log 2M)}{N_0}$, $\rho$ denotes the correlation coefficient between the branches, and it is defined as

$$\rho=\frac{E\left[(X-\bar{X})(Y-\bar{Y})\right]}{\left\{E[(X-\bar{X})]^2E[(Y-\bar{Y})^2]\right\}^{1/2}} \qquad (8)$$

where $\Gamma(\cdot)$ is the Gamma function, $D_{-V}(\cdot)$ is the parabolic cylinder function [4]. The validation of accuracy for (7) will be illustrated in Section IV with numerical analysis method.

### 2. The Coded Case

For the purpose of extending the analysis of the system performance to an *M*-ary radio system, the coded system is focused on the discussion of the linear binary block scheme. The BER performance of a binary block coded method is given as [8]

$$P_b=\frac{1}{n}\sum_{i=t+1}^{n}\beta_i\binom{n}{i}p^i(1-p)^{n-i} \qquad (9)$$

where $n$ is the necessary distance for coding, $\beta_i$ is the average error number reserved for correcting codeword which allows $i$ errors, and $t$ expresses the ability of error correction which can be taken as $t=\left[(d-1)/2\right]$, and $d$ denotes the minimum distance. However, the BER performance of MFSK with coded case can be determined by substituting the BER formula for uncoded shown in (7) into last equation, the $E_b$ will be replaced by $r_cE_b$, where $r_c$ denotes the coding rate of the coded MFSK. There is a special case, for example, the extended Golay code is chosen as (24, 12). The decision mentioned above is not the best fit for this case, but it can illustrate the features of the coded MFSK. The expand Golay codes shown in [8] are set as $\beta_4=4$, $\beta_5=8$, $\beta_6=120/19,\ldots$, $\beta_{24}=24$, etc. In order to maintain the assumption of channel is propagating over memoryless BSC (binary symmetric channel), the perfect bit interleaving for coding technique were adopted in this evaluation of the MFSK with coded case. For the situation of high $E_b/N_0$, which will be kept in the variant condition and the BER $P_b$ shown in (9) will be employed as inverse to power of the $m(t+1)$-th branch. Meanwhile, there is a simple multiplication for $m(t+1)$ and the fundamental channel integrated into a key code parameter. The accuracy of the BER performance for the MFSK coded case can be validated by comparing the results derived in (9) with the researched publication with the measurement data shown in [8].

## IV. NUMERICAL RESULTS

In this section, the BER performance of MPSK is analyzed by the numerical method with the Matlab package Fig. 1 and Fig. 2 illustrate the BER versus the SNR results for MFSK working in uncoded case with correlation coefficient $\rho$=0.5 and 0.9, respectively. It is known that the system BER performance for MFSK in uncoded case will become degradation when the branch number is increase. On the other hand, the BER performance variate with the characteristic of correlation between difference branches is shown in Figs. 3-4, where the correlation coefficient are set as $\rho$=0.1, 0.3, 0.5, 0.7 and 0.9. It can be easily understood that the performance is definitely degraded by the correlation coefficients, that is, the greater in $\rho$, the better in system performance. The performance of coded MFSK is discussed bellows. Fig. 5 and Fig. 6 show the BER versus to the SNR for the case of coded MFSK with the branch number $L$=2, 4, 8, 16, and, $\rho$=0.5 and $\rho$=0.9, respectively. The fact is same that the results shown for the uncoded MFSK case, that is, the BER performance depends on the branch number. The conditions assumed with branch numbers $L$=2, 8, and different correlation coefficients are expressed in Figs. 7-8, respectively. It is worthy to note that the trend of evaluated results of the coded cases shown in Figs. 7-8 are similar to the results illustrated in Figs. 3-4.

## V. CONCLUSION

In this paper, the BER performance of coded and uncoded MFSK working in the correlated Nakagami-$m$ fading channel is analyzed with different branch correlation coefficients. A new expression of BER performance with the parabolic cylinder function for the M-ary noncoherent modulation is derived. Some numerical results show the branch number and the correlation coefficient between the branches affect the system performance for the wireless radio system.

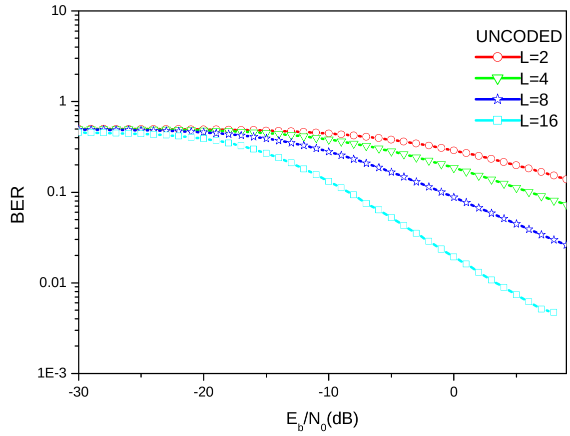


**Fig. 1. The BER vs SNR performance of uncoded MFSK with coefficient correlation $\rho$=0.5**

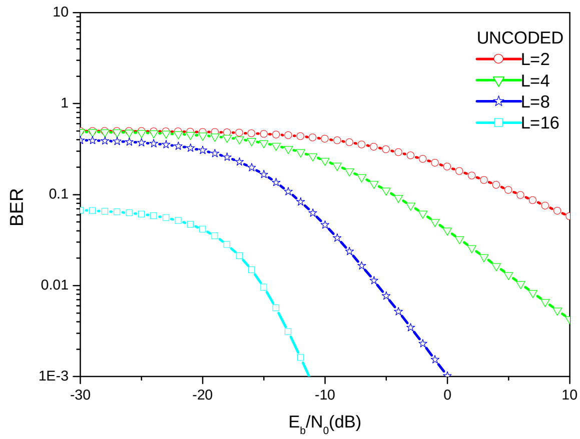


**Fig. 2. The BER vs SNR performance of uncoded MFSK with coefficient correlation $\rho$=0.9**

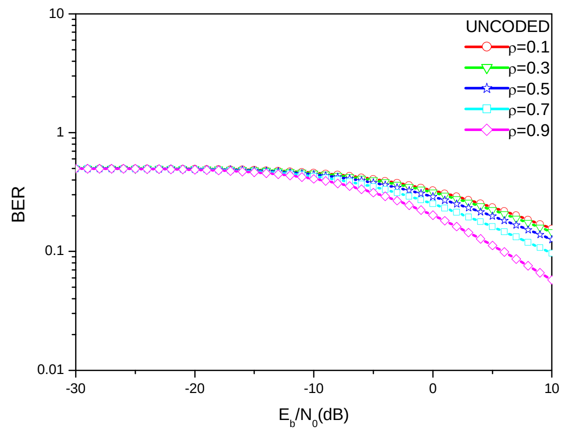


**Fig. 3. The BER vs SNR performance of uncoded MFSK with the branch counts $L$=2**

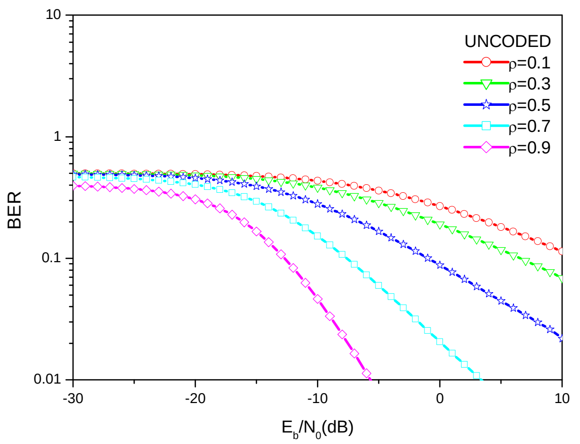


**Fig. 4. The BER vs SNR performance of uncoded MFSK with the branch counts *L*=8**

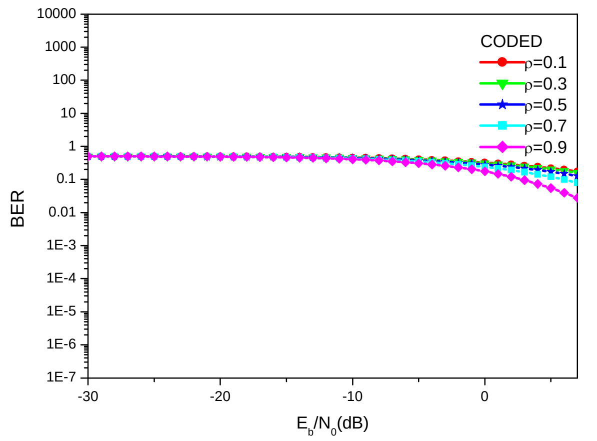


**Fig. 7. The BER vs SNR performance of coded MFSK with the branch counts *L*=2**

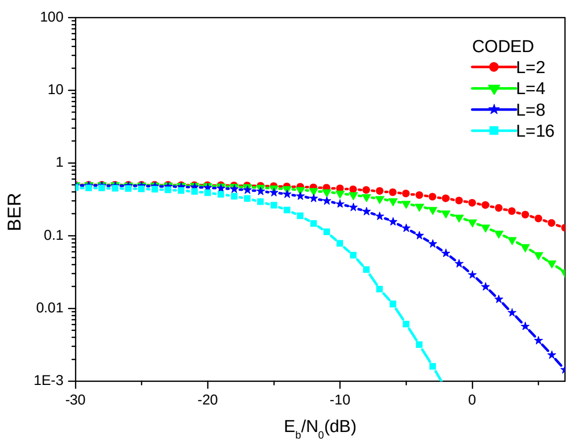


**Fig. 5. The BER vs SNR performance of coded MFSK with coefficient correlation $\rho$=0.5**

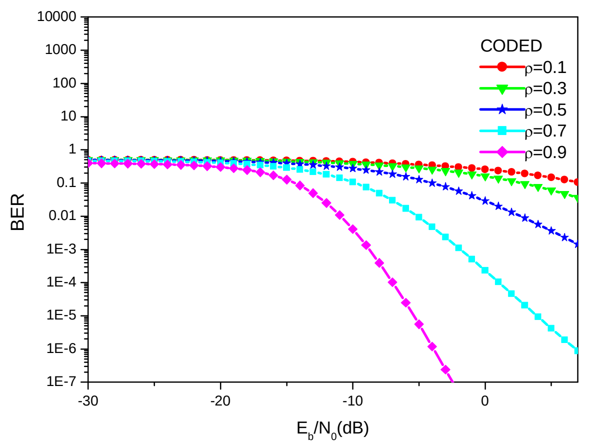


**Fig. 8. The BER vs SNR performance of coded MFSK with the branch counts *L*=8**

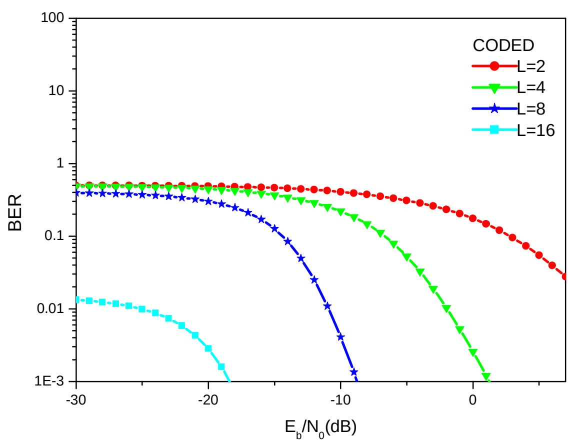


**Fig. 6. The BER vs SNR performance of coded MFSK with coefficient correlation $\rho$=0.9**

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## APPENDIX

In this appendix, the equation (7) was derived. It is known that the pdf of the fading amplitude is given as

$$p(R)=\frac{(1-\rho^2)^{L-1}}{\Omega}e^{-R/\Omega}\sum_{l=0}^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{R\rho^2}{\Omega(1-\rho^2)}\right)^l \tag{A1}$$

The average power was normalized as Ω=1, then the symbol error rate be calculated as

$$P_S=\int_0^\infty\sum_{i=1}^{M-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}e^{\frac{-1}{i+1}\frac{E_S R^2}{N_0}}p(R)dR \tag{A2}$$

where $P_b=\frac{M}{2(M-1)}P_s$, and $E_b=\frac{E_s}{\log 2M}$. By substitute (A1) into (A2), and the average bit error rate can be obtained as

$$P_b=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}\int_0^\infty e^{\frac{-i}{i+1}\frac{E_b(\log 2M)}{N_0}R^2}p(R)dR$$

$$=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}\int_0^\infty e^{\frac{-i}{i+1}\frac{E_b(\log 2M)}{N_0}R^2}\frac{(1-\rho^2)^{L-1}}{\Omega}e^{-R/\Omega}\sum_{l=0}^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{R\rho^2}{\Omega(1-\rho^2)}\right)^l dR.$$

$$=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}\frac{(1-\rho^2)^{L-1}}{\Omega}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{\Omega(1-\rho^2)}\right)\int_0^\infty e^{\frac{-i}{i+1}\frac{E_b(\log 2M)}{N_0}R^2}e^{-R/\Omega}R^l dR \tag{A3}$$

Some of the conditions are assumed as follows, the average power is assumed as Ω=1, and changing variable with

$$C_1=\frac{i}{i+1}\frac{E_b(\log 2M)}{N_0}$$

$$=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}(1-\rho^2)^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{1-\rho^2}\right)\int_0^\infty e^{-C_1R^2}e^{-R}R^l dR \tag{A4}$$

$$=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}(1-\rho^2)^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{1-\rho^2}\right)^l\int_0^\infty e^{-(C_1R^2+R)}R^l dR. \tag{A5}$$

By means of the formulas shown in [9], listed below as

$$\int_0^\infty x^{v-1}e^{-\beta x^2-\gamma x}dx=(2\beta)^{-v/2}\Gamma(v)\cdot\exp\left(\frac{\gamma^2}{8\beta}\right)D_{-v}\left(\frac{\gamma}{\sqrt{2\beta}}\right)\quad [\text{Re }\beta>0,\ \text{Re v}>0] \tag{A6}$$

where $D_{-v}(\cdot)$ is parabolic cylinder function, which is define as [4 ]

$$D_p(z)=\frac{e^{-\frac{z^2}{4}}}{\Gamma(-p)}\int_0^{\infty}e^{-xz-\frac{x^2}{2}}x^{-p-1}dx,\quad [\text{Re } p<0]$$

By setting the variables as $v=l+1$, and $p=-(l+1)$, $\beta=C_1=\frac{i}{i+1}\frac{E_b(\log 2M)}{N_0}$,

$$D_{-v}\left(\frac{1}{\sqrt{2\beta}}\right)=\frac{e^{-\frac{\frac{1}{2\beta}}{4}}}{\Gamma(l+1)}\int_0^{\infty}e^{-x\frac{1}{\sqrt{2\beta}}-\frac{x^2}{2}}x^{(l+1)-1}dx$$

$$=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}\left(1-\rho^2\right)^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{\left(1-\rho^2\right)}\right)^l$$

$$\times(2\beta)^{\frac{-(l+1)}{2}}\int_0^{\infty}e^{-x\frac{1}{\sqrt{2\beta}}-\frac{x^2}{2}}x^l dx \qquad \text{(A7)}$$

let $l=v-1$, $\gamma=1$, $C_1=\beta=\frac{i}{i+1}\frac{E_b(\log 2M)}{N_0}$, and put (A7) into (A3), the integral part can be obtained as

$$\int_0^{\infty}e^{-(C_1R^2+R)}R^l dR=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\times\frac{(-1)^{i+1}}{i+1}\left(1-\rho^2\right)^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{\left(1-\rho^2\right)}\right)^l$$

$$\times\left(2\beta\right)^{\frac{-(l+1)}{2}}\Gamma\left(l+1\right)e^{\left(\frac{1}{8\beta}\right)}D_{-v}\left(\frac{1}{\sqrt{2\beta}}\right), \qquad \text{(A8)}$$

and the BER shown in (A3) becomes as

$$P_b=\frac{M}{2(M-1)}\sum_{i=1}^{M-1}\sum_{l=0}^{L-1}\binom{M-1}{i}\frac{(-1)^{i+1}}{i+1}\left(1-\rho^2\right)^{L-1}\frac{\binom{L-1}{l}}{l!}\left(\frac{\rho^2}{\left(1-\rho^2\right)}\right)^l$$

$$\times(2\beta)^{\frac{-(l+1)}{2}}\Gamma(l+1)e^{\left(\frac{1}{8\beta}\right)}\frac{e^{-\frac{1}{8\beta}}}{\Gamma(l+1)}\int_0^{\infty}e^{-x\frac{1}{\sqrt{2\beta}}-\frac{x^2}{2}}x^l dx \qquad \text{(A9)}$$