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A Study on the Effect of Branch Correlation for MC-CDMA Systems under Different Configurations of Antenna Arrays

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ABSTRACT

A new exact expression of BER (bit error rate) and user capacity performance for MC-CDMA (multi-carrier coded-division multiple-access) communication systems is proposed in this study. Different antenna configurations, including triangular and linear configuration antenna arrays, are applied. In addition, the branch is assumed to be operating in correlated Nakagami-*m* fading environments. The CF (characteristic function) is adopted to solve the PDF (probability density function) of the SNR (signal-to-noise ratio) at the MRC (maximal ratio combining) output instead of other complicated analytical methods. Finally, the numerical results obviously show that the performance degradation of the MC-CDMA system is sensitive to the correlation of fading channels, which is validated by comparing the results shown in certain published research [8].

***Key Words*:** MC-CDMA, correlated Nakagami-*m*, characteristic function, triangular configuration antenna, linear configuration antenna

不同的天線配置架構下分支相關效應影響 MC-CDMA 系統效能之分析研究

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摘　要

本篇論文提出新的多載波分碼多重近接（multi-carrier coded-division multiple-access, MC-CDMA）系統的位元錯誤率（bit error rate, BER）與使用者容量之效能表示式，其中應用了包括三角式（triangular）與線性式（linear）結構的天線配置；除此之外，亦假設通訊通道分支是工作於呈現具有相關性之 Nakagami-*m* 分布的衰落環境之中。推導過程中。採取計算特徵函數（characteristic function, CF）的方式，以避開利用其他複雜的分析方法，藉以求得接收端輸出訊雜比（signal-to-noise ratio, SNR）的機率密度函數（probability density function, pdf）。最後經由數值分析之結果顯示，MC-CDMA 系統之效能確實會受制於不同之天線的配置方式與衰落通道的相關性所影響。

關鍵詞：多載波分碼多重近接系統，相關性 Nakagami-*m* 分佈，特徵函數，三角式天線，線性式天線

I. INTRODUCTION

The advantages of spectrum efficient, interference immune, high data transmission rate, and insensitivity to frequency selective channel become some of the reasons for high-speed development of spread spectrum technical. It is also known that the multiple access system bases on direct sequence CDMA (code-division multiple-access) have drawn recent interest in the application of wireless radio system [8]. Especially, multi-carrier CDMA (MC-CDMA) appears to be a considerable candidate for mobile radio communication system in the future.

The BER (bit error rate) analysis of MC-CDMA based on considering different kinds of assumptions, so far, have been dedicated in numerous previously researches [2, 8, 11]. The performance evaluation of MC-CDMA over multipath fading channels was studied in [11]. The results presented in [2] are for uplink channel using MRC (maximal ratio combining) with the assumed frequency offsets condition in correlated fading. The performance of MC-CDMA in non-independent Rayleigh fading was studied in [4]. In [3], which by use of the method of CF (characteristic function) and residue calculation for downlink MC-CDMA. Both of the envelopes and phases correlation are considered in [5] to evaluate the performance of a MC-CDMA system operates in Rayleigh fading channel. The literature in [9] illustrated the error probability for MC-CDMA systems assumed that the transmission channel is in Nakagami-m fading, and the postdetection of EGC (equal gain combining) is considered.

Thought the ideal of correlation between any pair of branch is going to impact the performance of the wireless communication system, there is a new generic expression of BER performance for MC-CDMA system is evaluated in this paper. The transmission channels with correlated Nakagami-*m* fading distribution are considered as a reasonable assumption while the separation of transmitter and receiver is less than 5λ, where λ is wavelength [10]. We adopted CF (characteristic function) to solve the pdf (probability density function) of the SNR (signal-to-noise ratio) at the MRC (maximal ratio combining) output instead of the other complicate analysis methods. The numerical results do show that the channel correlation coefficient affect the performance of MC-CDMA systems.

The rest of this paper is organized as follows. Section II gives a description of the MC-CDMA system model includes the transmitter, the receiver and the correlated Nakagami-*m* channel model. The performance analysis for the MC-CDMA system is carried out in section III. There are numerically results shown in section V. Finally, section VI draws a briefly conclusion.

II. SYSTEM MODEL OF MC-CDMA SYSTEM

1. Transmitter Model

The MC-CDMA system model is described in this section. It is assumed that there exist K simultaneously users with N subcarrier within a signal cell. Any effect of correlation among users is going to be ignored by assuming the number of users is uniformed of distribution. As shown in Fig.1, a signal data symbol is replicated into N parallel copies. The signature sequence chip with a spreading code of length L is used to BPSK (binary phase shift keying) modulated each of the N subscrier of the k-th user. Where the subcarrier has frequency F/T_b Hz, and where F is an integer number [8, 11]. The technical described above is same as to the performance of OFDM (orthogonal frequency division multiplexing) on a direct sequence spread-spectrum signal when set F=1. The larger values of F, the more transmit bandwidth increase. The transmitted signal the resulting transmitted baseband signal $S^{(k)}(t)$ corresponding to the M data bit size can be expressed as

$$S^{(k)}(t) = \sqrt{\frac{2P}{N}} \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} a^{(k)}[n] b^{(k)}[m] P_{T_b}(t) \times \mathrm{Re}[e^{j\varpi_n t}], \begin{cases} a^{(k)}[n] \in \{-1,1\} \\ b^{(k)}[m] \in \{-1,1\} \end{cases} \quad (1)$$

where P is the power of data bit, M denotes the number of data bit, N denotes the number of subcarriers, the sequencer $a^{(k)}[0],\ldots,\ a^{(k)}[N\text{-}1]$ and $b^{(k)}[0],\ldots,\ b^{(k)}[M\text{-}1]$ represent the signature sequence and the data bit of the performance of MC-CDMA systems for the k-th user, respectively. The $P_{T_b}(t)$ is defined as an unit amplitude pulse that is non-zero in the interval of [0, T_b], and Re[$\cdot$] denotes the real part of a

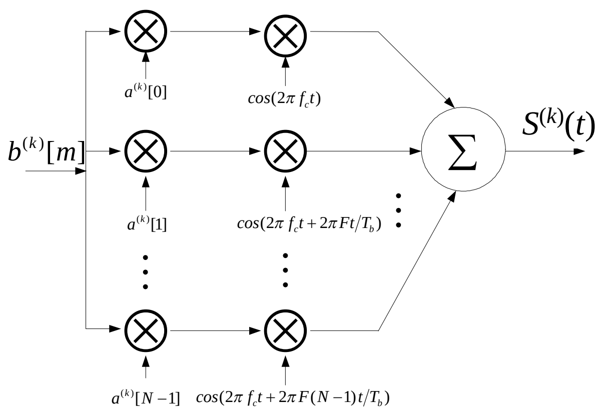

Fig. 1 The transmitter model of the MC-CDMA system

complex number, $\omega_n=2\pi(f_c+nF/T_b)$ is the angular frequency of the n-th subcarrier.

A frequency-selective channel with $1/T_b \ll BW_c \ll F/T_b$ is addressed in this paper, where BW_c is the coherence bandwidth. This channel model means that each modulated subcarrier does not experience significant dispersion and with transmission bandwidth of $1/T_b$, i.e. $T_b \gg T_d$, where $1/T_d$ is the Doppler shift typically in the range of 0.3~6.1 Hz in the indoor environment [8], and the amplitude and phase remain constant even the symbol duration T_b. In addition to, the channel of interest has the transfer function of the continuous-time fading channel assumed for the k-th user can be represented as

$$H^{(k)}[f_c + i\frac{F}{T_b}] = \beta_i^{(k)} e^{j\theta_i^{(k)}} \tag{2}$$

where $\beta_i^{(k)}$ and $\theta_i^{(k)}$ are the random amplitude and phase of the channel of the k-th user at frequency $f_c+i(F/T_b)$. In order to follow the real world care, the random amplitude, $\beta_i^{(k)}$ are assumed to be a set of N correlated not necessarily identically distributed.

2. Channel Model

In this paper, a slowly varying fading channel is considered, that is, the channel parameters are unchanged over one bit duration T_b. The equal fading severities are considered for all of the channels, namely $m_i=m$, $i=1,\ldots,N$. The pdf of the fading amplitude for the k-th user *with* i-th channel, $\beta_i^{(k)}$, are assumed as r. v. (random variable) with the Nakagami-m distribution, and given as [6]

$$P(\beta) = \frac{2\beta^{2m-1}}{\Gamma(m)} \cdot (\frac{m}{\Omega})^m \cdot \exp(-\frac{m\beta^2}{\Omega}), \qquad \beta \ge 0 \tag{3}$$

where $\Gamma(\cdot)$ is the gamma function defined by $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$, $\Omega = E[\beta^2]$ denoting expectation, the parameter m of the amplitude distribution characterizes the severity of the fading, and it is defined as

$$m = \frac{\Omega^2}{E[(\beta^2 - \Omega)^2]} \ge 0.5 \tag{4}$$

It is well known that $m=0.5$ (one-sided Gaussian fading) corresponds to worst case fading condition, $m=1$ and $m=\infty$ correspond to Rayleigh fading (purely diffusive scattering) and the non-fading condition, respectively. The sum of N-correlated gamma variable will present at the output of the receiver, that is, Let $[\gamma_i]$, $i=1,\ldots,N$ be a set of N correlated identically distributed, and all the figure parameters and the average power are assumed equivalent, that is, $m_i=m_j=m$, and $\Omega_i=\Omega_j=\Omega$, where $i \neq j$, for i, $j=1,\ldots,N$. The power at the output of the MRC is a function of the sum of the squares of signal strengths, and is given as

$$R = \sum_{i=1}^{N} \gamma_i \tag{5}$$

Then, the joint characteristic function of the instantaneous SNR can be obtained as [7]

$$\varphi_r\left[j(u_1 \ldots u_N)\right] = E\left[\exp(\sum_{t=1}^{N} j\gamma_t u_t)\right] = \det\left[I - jTA\right]^{-m} \tag{6}$$

where I is the unit matrix, the fading parameter is assumed to be identical for all branch, A denotes the positive definite matrix determined by the branch covariance matrix C_r. Let $s=ju$, and the last equation can be expressed as begin with the partial expression of CF and is written as

$$\varphi_r(s) = \det[I - sA]^{-m} = \prod_{t=1}^{N} \frac{1}{(1 - s\lambda_t)^m} \tag{7}$$

where λ_t represents the eigenvalues of the matrix A. All the eigenvalues are assumed distinct. The equation (7) can be expressed as a partial sum

$$\det[I - sA]^{-m} = \sum_{l=1}^{N} \sum_{r=1}^{m} \varepsilon_{lr}(1 - s\lambda_l)^{-r} \tag{8}$$

where the coefficients ε_{lr} can be determined in two cases, that is given as

$$\varepsilon_{lm} = \prod_{t=1, t \neq l}^{L} \frac{1}{(1 - s\lambda_t)^m}\Bigg|_{s=(\lambda_l)^{-1}} \tag{9}$$

For $r=m$, and

$$\varepsilon_{lr} = \frac{1}{(m-r)!(-\lambda_l)^{m-r}} \frac{d^{m-r}}{ds^{m-r}} \prod_{t=1, t \neq l}^{L} \frac{1}{(1 - s\lambda_t)^m}\Bigg|_{s=(\lambda_l)^{-1}} \tag{10}$$

When consider that each term in (8) corresponds to a Gamma distribution, the pdf of the sum as shown in (5), R, can be determined as the linear combination of the distribution of mN independent gamma random variable. Thus the pdf in terms of the CF of $(1-s\lambda_l)^{-r}$ can be expressed as

$$p_R(t)=(\frac{t}{\lambda_l})^{r-1}e^{(-t/\lambda_l)}\frac{1}{\lambda_l\Gamma(r)} \quad (11)$$

and, the distribution for variable R can be calculated as

$$f_R(t)=\sum_{l=1}^{N}\sum_{r=1}^{m}\varepsilon_{lr}P_R(t) \quad (12)$$

3. Receiver Model

For K active transmitters, the received signal $r(t)$ can be written as

$$r(t)=\sqrt{\frac{2P}{N}}\sum_{k=0}^{K-1}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\beta_{m,n}a^{(k)}[n]b^{(k)}[m]$$

$$\times P_{T_b}(t-mT_b-\tau^{(k)})\cos(\omega_n t+\theta_{m,n})+n(t) \quad (13)$$

where $n(t)$ is the AWGN (additive white Gaussian noise) with a double-sided power spectral density of $N_0/2$. We can evaluate the local-mean power, $P_n^{(k)}$, which is given as

$$P_n^{(k)}=E[\left(\beta_n^{(k)}\right)^2]\cdot\frac{\rho}{N} \quad (14)$$

The total-mean power of the k-th user is defined to be $P_k=N\cdot P_n^{(k)}$, if the local-mean power of the subcarriers is assumed equal. Assuming that acquisition has been accomplished for the user of interesting (k=0). In addition, the system operates synchronously with each user having the same clock is assumed, and the MRC diversity reception technique is considered in this paper. For the reason of using MRC, it is assumed that perfect phase correction can be obtained, i.e. $\hat{\theta}_{0,i}=\theta_{0i}$. Demodulating each subcarrier includes applying a phase correction, $\hat{\theta}_{0,i}$, and a gain correction factor $d_{0,n}=\beta_{0,n}\cdot\alpha_0[n]$ is multiplied by the n-th subcarrier signal.

With all the assumptions for MRC diversity, the decision variable D_0 of the m-th data bit of the reference user, can be written as

$$D_0=\frac{1}{T_b}\int_{mT_b}^{(m+1)} r(t)\cdot\sum_{n=0}^{N-1}a_0[n]\cdot d_{0,i}\cdot\cos(\omega_n t+\theta_{0,0})dt$$

$$=U_s+I_{MAI}+\eta_0 \quad (15)$$

where $r(t)$ is the received signal as shown in (13), $d_{0,i}$ is the gain factor for MRC diversity, and the three terms in the second equivalent of the equation (15) are going to be described as follows. The first term represents the desired signal, can be expressed as

$$U_s=\sqrt{\frac{P}{2N}}\sum_{n=0}^{N-1}\beta_{0,i}^2\cdot a_0[m] \quad (16)$$

and the second term, I_{MAI}, is the MAI (multiple access interference) contributed from other users, which can be determined as

$$I_{MAI}=\sqrt{\frac{P}{2N}}\cdot\sum_{k=1}^{K-1}\sum_{n=0}^{N-1}a_k[m]\cdot b_k[m]\cdot a_0[m]$$

$$\times\beta_{k,m}\cdot\beta_{0,m}\cdot\cos(\theta'_{k,n}) \quad (17)$$

where $\theta'_{k,n}=\theta_{0,n}-\theta_{k,n}$ and $\theta_{k,n}$ are *i.i.d* (identically independent distributed) r.v's, uniformly distributed over $[0,2\pi)$. The variance of MAI condition on $\beta_{0,n}$ need to be determined and can be obtained as

$$\sigma_{I_{MAI}}^2=\frac{P(K-1)}{6N}E[\beta_{k,m}^2]\cdot\sum_{n=0}^{N-1}\beta_{0,n}^2 \quad (18)$$

The third term, η_0 is corresponds to the AWGN term and with the variance given as

$$\sigma_{\eta_0}^2=\frac{N_0}{4T_b}\sum_{n=0}^{N-1}\beta_{0,n}^2 \quad (19)$$

III. PERFORMANCE ANALYSIS

For BPSK (binary phase shift keying) modulation the average BER can be obtained by averaging the conditional BER over the joint pdf of the instantaneous SNR value corresponding to the N subcarrier. Furthermore, let's define the total normalized power of the reference user is

$$P_t=\frac{1}{\Omega}\sum_{n=0}^{N-1}(\beta_n^{(0)})^2 \quad (20)$$

Hence the received SNR at the output of the receiver of the reference user, $snr_n^{(0)}$, may be computed and expressed as the impact form [1]

$$snr_n^{(0)} = \frac{S^2}{\sigma_\eta^2 + \sigma_I^2} = P_t \cdot \sum_0 \tag{21}$$

where the subscript *0* represents the reference user, and the average received *SNR* per bit can be obtained as

$$\sum{}_0 = \frac{1}{\frac{2(K-1)}{3} + N\xi} \tag{22}$$

where $\xi = 1 \Big/ \frac{E_b}{\Omega N_0}$, and $\Omega = (\beta_n^{(k)})^2$ denotes the average power of *n*-th carrier of *k*-th user. Hence the average BER is calculated as

$$P_{er} = \int_0^\infty 0.5 f_R(t) \cdot erfc[snr_n^{(0)}]dt \tag{23}$$

where $f_R(x)$ expresses the conditional pdf of *R* is shown is (12), and $erfc(\cdot)$ is the complementary error function. By use of changing variables, let $P_t \cdot \sum_0/2$=u, hence the equation (23) can be obtained by applying the results of [6] and becomes as

$$P_{er} = \sum_{l=1}^{L}\sum_{r=1}^{m} \varepsilon_{lr} \left(\frac{1-\mu_l}{2}\right)^r \frac{2}{\sum 0} \sum_{k=0}^{r-1} \binom{r-1-k}{k} \left(\frac{1+\mu_l}{2}\right)^k \tag{24}$$

where $\mu_l = (2/2 + \lambda_l \cdot \sum{}_0)^{0.5}$ which is defined as in [6]. In this paper, only the integer *m* values are considered.

IV. NUMERICAL RESULTS

The performance results derived in last section will be numerically evaluated. Moreover, there are two kinds of antenna configuration scenarios include triangular and linear array will be considered in this numerical results. It is well known that both of the array configurations and the incident angle of the incoming signal will decide the correlation matrix of an antenna array. The influence of the fading parameter on the error performance using linear and triangular arrays will be assumed. The separation between the antennas will be set as $d=8\lambda=9.26ft$, such that the correlation matrix of an antenna array with linear and triangular configuration can be obtained as, respectively [10]

$$C_r = \begin{bmatrix} 1 & 0.727 & 0.913 \\ 0.727 & 1 & 0.913 \\ 0.913 & 0.913 & 1 \end{bmatrix} \tag{25}$$

for the linear array, and

$$C_r = \begin{bmatrix} 1 & 0.795 & 0.605 \\ 0.795 & 1 & 0.795 \\ 0.605 & 0.795 & 1 \end{bmatrix} \tag{26}$$

for the triangular array. Where note that only the correlation matrix of the linear array has a Toeplitz structure.

In the numerical analysis, the plots of SNR versus BER performance for the linear array and the triangular configuration are shown in Fig. 2 and Fig. 3, respectively, in which the different user number, *k=4*, *12, 32, 64,* the fading parameter, *m=3*, and the number of subcarrier, *N=3*, are assumed. It is reasonable to explain that the BER will become much degrade while the user number is increase gradually. We study the difference of the SNR versus BER performance between the linear array and triangular array configuration with different user number in Fig. 4. The matrix defines the covariance matrix for linear and triangular array are shown in (25) and (26), respectively. Fig. 5 illustrates the results from the effect of different of fading parameters, *m=2,3,4,5,* subcarrier is set as *N=3*, and the results shows that the better the system performance the more the fading parameters is increase.

V. CONCLUSIONS

The effects of correlated fading channel with Nakagami distribution are investigated in this paper. There are some closed-forms for BER performance derived with the consideration of different antenna array configuration, that is, linear array and triangular array configuration. The results show that the correlation between fading channels are not negligible.

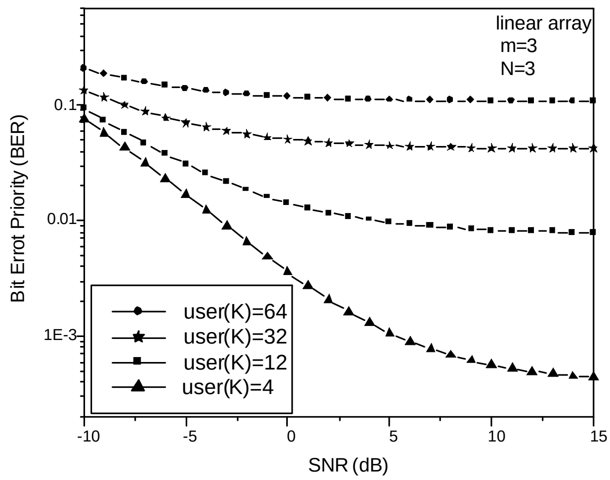

Fig. 2. SNR vs. BER plots for linear array configuration with *m=3*, and *N=3*

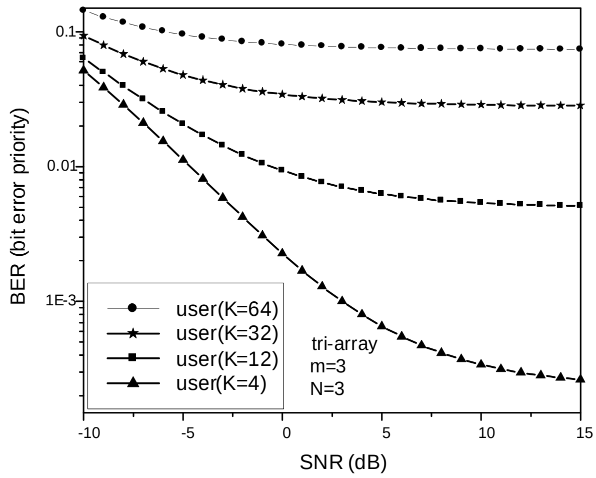

Fig. 3. SNR vs. BER plots for triangular array configuration with *m=3*, and *N=3*

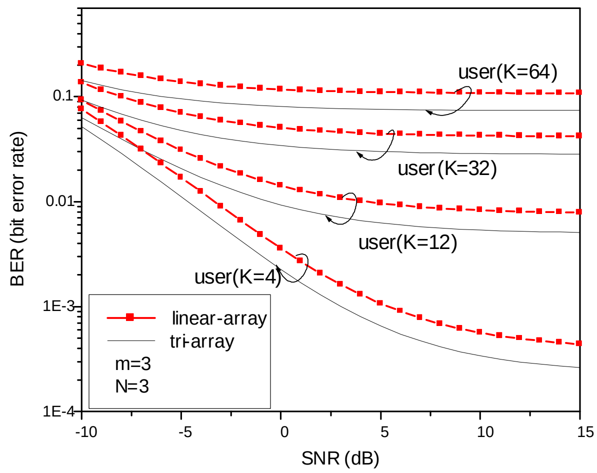

Fig. 4. SNR vs. BER plots for different antenna array configuration with *m=3*, and *N=3*

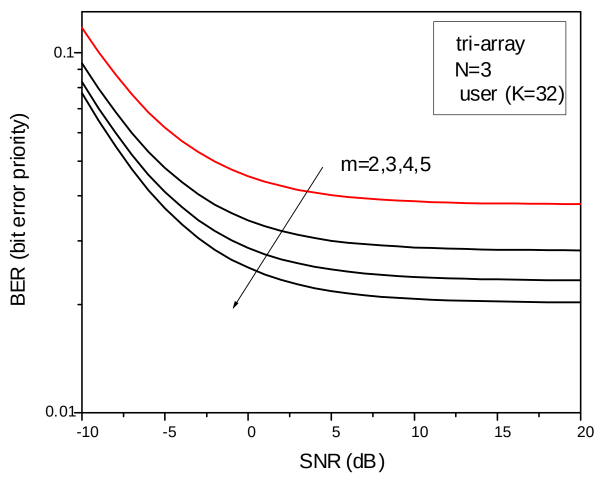

Fig. 5. SNR vs. BER plots for triangular array configuration with different fading parameters, *N=3*, and *K=32*

REFERENCES

1. Chen, J. I. Z. and R. K. S. Miao (2004) Performance evaluation of MRC for MC-CDMA communication system over Nakagami-m channels. Proceeding of 4th International Symposium on CSNDSP, 180-283.
2. Efthymoglou, G. P., V. A. Aalo and H. Helmken (1997) Performance analysis of coherent DS-CDMA systems in a Nakagami fading channel with arbitrary parameters. *IEEE Translation on Vehicular Technology*, 46(2), 289-297.
3. Gradshteyn, I. S. and I. M. Ryzhik (2000) *Table of Integrals, Series, and Products*, 6th Ed., Academic Press, New York, NY.
4. Lee, W. C. (1993) *Mobile Communications: Design Fundamentals*, 2nd Ed., 202-211. Wiley, New York, NY.
5. Nakagami, M. (1960) The m-distribution- a general formula of intensity distribution of rapid fading. In: *Statistical Methods in Radio Wave Propagation*, 3-36. Pergamon, Oxford, U.K.
6. Proakis, J. (1995) *Digital Communications*, 3rd Ed., 47-48. McGraw Hill, New York, NY.
7. Schwartz, M., W. R. Bennett and S. Stein (1996) *Communication Systems and Techniques*, Inter-University Electronic Series, 4, McGraw-Hill, New York, NY.
8. Zexian, Li and M. Latva-aho (2002) Analysis of MRC receivers for asynchronous MC-CDMA with channel estimation errors. IEEE 7th International Symposium on Spread-Spectrum Technology & Applications, Czech Republic.
9. Zexian, Li, and M. Latva-aho (2002) Error probability for MC-CDMA in Nakagami fading channels using equal gain combining. IEEE International Conference on Communications, New York, NY.
10. Zhang, Q. T. (1999) Maximal-ratio combining over Nakagami fading channels with an arbitrary branch covariance matrix. *IEEE Translation on Vehicular Technology*, 48(4), 1141-1150.
11. Zhang, Q. T. (2003) A generic correlated Nakagami fading model for wireless communications. *IEEE Translation on Communications*, 51(11), 1745-1748.

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