

# A New Approach to Decentralized Adaptive Output Feedback Controller Design for Mismatched Uncertain Large-scale Systems Using Linear Matrix Inequalities

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## ABSTRACT

In this study, an adaptive sliding-mode controller was designed for use in a class of large-scale systems that exhibit mismatched uncertainties and exogenous disturbances. First, a new sliding mode controller was proposed using only output variables. Adaptation laws were developed for calculating the unknown upper bounds of mismatched uncertainties; these updated values were used to establish a class of decentralized adaptive output feedback controllers. Both sliding surfaces and adaptive sliding-mode controllers can be easily attained using a linear matrix inequality technique. Moreover, a stability analysis was conducted to assess the overall system. Finally, a numerical example was used to demonstrate the efficacy of the proposed method.

**Key Words:** large-scale systems, decentralized adaptive controller, linear matrix inequalities (LMI), sliding mode control (SMC).

## 以線性矩陣不等式理論設計非匹配不確定大型系統之 分散式自適應輸出回授控制器

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## 摘要

本文對於具有非匹配不確定成分和外部擾動的大型系統，設計其自適應滑動模式控制器，新的自適應動態滑動模式控制器僅僅使用輸出變數。本文提供一個自適應法則來計算非匹配不確定成分的未知上界，並使用更新值來完成此種分散式自適應輸出回授控制器的設計。另外，通過使用線性矩陣不等式 (LMI) 以及凸集最佳化技術，滑動平面和自適應滑動模式控制器得以容易實現。此外，本文也完成整個系統的穩定性分析。最後，一個數值例子用來證明本方法的正確有效。

**關鍵詞：**大型系統，分散自適應控制，線性矩陣不等式，滑動模式控制。

## 1. Introduction

The theory of variable structure control (VSC) is often used in controlling of uncertain systems. The design procedure generally involves two main steps: firstly, the selection of a sliding surface which induces a stable reduced-order dynamics assigned by the designer, and secondly the synthesis of a switching control law to force the closed-loop system trajectories onto and subsequently remain on the sliding surface.

Interconnected systems are characterized by large number of variables, strong interaction between system variables and the complex structure. In Lee [13], a very useful method of stabilization sliding mode control is suggested for the large-scale system. In this method each isolated subsystem is stabilized by a local output feedback control, and a global control is chosen to minimize the effects of interconnections between the subsystems. The work [18] introduced the sliding mode control to mismatched uncertain large-scale systems. They have given sufficient solution for the system in the sliding mode which is completely invariant to both matched and mismatched uncertainties. Control strategies are inherently robust with respect to a wide variety of perturbations in the interconnections. The strategies can be involving individual subsystems.

On the other hand, a major drawback of VSC is that the state variables have to be accessible. In many practical systems, the state variables are not accessible for direct measurement or the number of measuring devices is limited. In this situation, there are two approaches in designing the sliding mode output feedback controllers. One is to use state observers to provide an estimate of the unmeasured states [6, 12, 14, 19] and [22]. The other is to utilize the output-based controllers, such as static gain and dynamic compensator types [18] and [28]. Thus, the design of asymptotic observers and dynamic compensators are very important and has been established [2-4] and [31]. However, the direct output feedback design in variable structure system (VSS) is worth investigating. Zak and Hui [29] proposed some basic and important results of static output feedback VSC. Two major limitations in the paper [13] are solved by Kwan [10] that used a dynamic output feedback VSC for single-input/single-output (SISO) VSS. The work [17] proposed an output feedback controller for mismatched uncertain VSS. Another further result of output feedback VSC

is also proposed by [11], which is to discuss the controllers design for multi-input/multi-output (MIMO) VSS.

Recently, There have presented some decentralized adaptive control schemes for large-scale systems, see, e.g., [8], [21] and [22]. In [22], not only closed-loop stability, but also decentralized asymptotic tracking is achieved. On the other hand, to cancel the effect of the interconnections on tracking performance, [6] proposed a decentralized adaptive output feedback control for large-scale interconnected systems using a linearly parameterized neural. In order to solve the unknown high-frequency gain signs [25] and [27] presented a decentralized adaptive control scheme for large-scale non-linear systems. In [23] is devoted to decentralized output-feedback adaptive control for a class of uncertain interconnected non-linear systems which is able to eliminate the explosion of complexity problem inherent in traditional backstepping design. In [30] proposed a scheme to design decentralized backstepping adaptive tracking controllers for a class of nonlinear interconnected systems in the presence of external disturbances which the transient tracking error performance can be adjusted by choosing suitable design parameters. In [26], the more difficult problem of adaptive control with time delays in system inputs is also considered. However, as stated in [23], the assumption on the knowledge of the high-frequency-gain sign does not appear to be realistic in the general case. A decentralized model reference adaptive control scheme is proposed in [15] where the interconnections considered are linear and matched. While backstepping approaches in [14, 19, 22-23, 25, 27] and [30] require the nominal systems have a special structure and [6, 12] can not apply for systems with mismatched uncertainty in state matrix.

Motivated by the previous works, in this paper, we attempt to address a decentralized adaptive output feedback control for a class of matched and mismatched uncertain large scale systems with unmeasurable states.

The main contributions of this paper lie in the following.

- 1) A new approach is developed to design adaptive sliding-mode controllers using only the output information and a new adaptive law to solve upper bounded mismatched uncertainty for a class of large scale systems.
- 2) By using linear-matrix inequalities (LMIs), both sliding surfaces and adaptive sliding-mode controllers can be easily achieved via a convex optimization technique. Our approach

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does not need the availability of the state variables so that our method is very useful and more realistic since it can be easily implemented in practice.

## 2. Problem Formulations and Preliminaries

In this paper, we consider a class of mismatched uncertain large-scale systems that is decomposed into  $L$  subsystems. The state space representation of each subsystem is described by the following equation:

$$\begin{aligned} \dot{x}_i &= (A_{ii} + \Delta A_{ii})x_i + B_i(u_i + \xi_i(x_i, t)) + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ij}x_j \\ y_i &= C_i x_i \end{aligned} \quad (1)$$

where  $x_i \in R^{m_i}$ ,  $u_i \in R^{m_i}$ ,  $y_i \in R^{p_i}$  are the state variables, control inputs and outputs of the subsystem, respectively.  $A_{ii}$ ,  $B_i$  and  $A_{ij}$  are constant matrices with appropriate dimensions.

The matrix  $\Delta A_{ii}x_i$  represents the mismatched uncertainty which the matching condition is not satisfied. The matrix  $B_i \xi_i(x_i, t)$  represents the matched uncertainty of the system.

In order to design the new adaptive output feedback controller, we denote the sliding surface by  $\sigma_i = 0$ ,  $i = 1, 2, \dots, L$ , where the sliding functions

$$\sigma_i = [-N_i \quad K_i] C_{i2}^{-1} y_i \quad (2)$$

are  $m_i$  state vector matrix with  $K_i \in R^{m_i \times m_i}$  is nonsingular and  $N_i \in R^{m_i \times (p_i - m_i)}$ . The sliding mode is defined by  $\sigma_i(x_i, t) = 0$  and  $\dot{\sigma}_i(x_i, t) = 0$ . From equation (2), one can see that there are only output variables used.

Here we make the following assumptions

**Assumption 1:** All the pairs  $(A_{ii}, B_i)$  are stabilizable.

**Assumption 2:** The input matrix  $B_i$  and  $C_i$  have full rank matrix with  $m_i < p_i < n_i$ .

**Assumption 3:** There exist a known non-negative constant

$k_{m_i}$  such that for  $i, j=1, 2, \dots, n$ ,  $\|\xi_i(x_i, t)\| \leq k_{m_i} \|x_i(t)\|$ .

**Assumption 4:**  $\Delta A_{ii} = D_{ii} F_{ii}(x_{ii}, t) E_{ii}$  where  $F_{ii}(x_{ii}, t)$  is unknown but bounded  $\|F_{ii}(x_{ii}, t)\| \leq 1$ , and  $D_{ii}, E_{ii}$  are known matrices of appropriate dimensions.

With the above assumptions, the problems considered in this paper can be formulated as following

- 1) Define a new sliding function using only output variables.
- 2) Sufficient conditions are developed base on LMI technique and using a Lyapunov approach such that the sliding motion is asymptotically stable.
- 3) Design an adaptive controller to solve upper bounded of mismatched uncertainty for a class of large scale systems.
- 4) The controller guarantee that the state trajectories of the mismatched uncertain large scale systems (1) are in the sliding mode in finite time and stay on it thereafter.

## 3. Sliding-Mode Decentralized Output Feedback Controller Design

In this section, we design a decentralized adaptive output feedback controller for the system (1). The first, we select an appropriate sliding surface, which should have the property that the desired performance can be achieved by only using output information. The second, we organize a decentralized adaptive controller that forces the system state to reach the sliding surface in a finite amount of time and stay on it thereafter.

### 3.1 Design sliding function using only output information

Since  $\text{rank}(C_i B_i) = m_i$ , then from [31] by the state transformation  $\bar{z}_i = T_i x_i$  such that system (1) has following regular form.

$$\begin{aligned} \dot{\bar{z}}_i &= \begin{bmatrix} A_{ii1} & A_{ii2} \\ A_{ii3} & A_{ii4} \end{bmatrix} + \begin{bmatrix} D_{ii1} \\ D_{ii2} \end{bmatrix} F_{ii} \begin{bmatrix} E_{ii1} & E_{ii2} \end{bmatrix} \bar{z}_i \\ &+ \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} (u_i + \xi_i) + \sum_{\substack{j=1 \\ j \neq i}}^L \begin{bmatrix} A_{ij1} & A_{ij2} \\ A_{ij3} & A_{ij4} \end{bmatrix} \bar{z}_j \\ y_i &= \begin{bmatrix} 0 & C_{i2} \end{bmatrix} \bar{z}_i \end{aligned} \quad (3)$$

where  $T_i A_{ii} T_i^{-1} = \begin{bmatrix} A_{ii1} & A_{ii2} \\ A_{ii3} & A_{ii4} \end{bmatrix}$ ,  
 $T_i D_{ii} F_{ii} E_{ii} T_i^{-1} = \begin{bmatrix} D_{ii1} \\ D_{ii2} \end{bmatrix} F_{ii} \begin{bmatrix} E_{ii1} & E_{ii2} \end{bmatrix}$ ,  $T_i A_{ij} T_j^{-1} = \begin{bmatrix} A_{ij1} & A_{ij2} \\ A_{ij3} & A_{ij4} \end{bmatrix}$ ,  
 $T_i B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix}$  and  $C_i T_i^{-1} = \begin{bmatrix} 0 & C_{i2} \end{bmatrix}$ .  $B_{i2} \in R^{m_i \times m_i}$  and  
 $C_{i2} \in R^{p_i \times p_i}$  are nonsingular.

Letting  $\bar{z}_i = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix}$  where  $z_{i1} \in R^{n_i - m_i}$ ,  $z_{i2} \in R^{m_i}$

$$\begin{aligned} \dot{z}_{i1} &= (A_{ii1} + D_{ii1} F_{ii} E_{ii1}) z_{i1} + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ij1} z_{j1} \\ &+ (A_{ii2} + D_{ii2} F_{ii} E_{ii2}) z_{i2} + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ij2} z_{j2} \end{aligned} \quad (4)$$

$$\begin{aligned} \dot{z}_{i2} &= (A_{ii3} + D_{ii3} F_{ii} E_{ii3}) z_{i1} + B_{i2} (u_i + \xi_i) \\ &+ (A_{ii4} + D_{ii4} F_{ii} E_{ii4}) z_{i2} \\ &+ \sum_{\substack{j=1 \\ j \neq i}}^L (A_{ij3} z_{j1} + A_{ij4} z_{j2}) \end{aligned} \quad (5)$$

Obviously, the system (4) represents the sliding-motion dynamic of system (3). Hence, the corresponding sliding function can be chosen as follows:

$$\sigma_i = \begin{bmatrix} -N_i & K_i \end{bmatrix} C_{i2}^{-1} y_i \quad (6)$$

where matrix  $K_i \in R^{m_i \times m_i}$  is nonsingular and  $N_i \in R^{m_i \times (p_i - m_i)}$  will be selected late.

Thus, substituting  $y_i = \begin{bmatrix} 0 & C_{i2} \end{bmatrix} \bar{z}_i$  into the equation (6), we have

$$\begin{aligned} \sigma_i(t) &= \begin{bmatrix} -N_i & K_i \end{bmatrix} \begin{bmatrix} 0_{p_i \times (n_i - p_i)} & I_{p_i \times p_i} \end{bmatrix} z_i \\ &= \begin{bmatrix} -N_i & K_i \end{bmatrix} \begin{bmatrix} 0_{(p_i - m_i) \times (n_i - p_i)} & I_{(p_i - m_i) \times (p_i - m_i)} & 0_{(p_i - m_i) \times m_i} \\ 0_{m_i \times (n_i - p_i)} & 0_{m_i \times (p_i - m_i)} & I_{m_i \times m_i} \end{bmatrix} z_i \\ &= \begin{bmatrix} -N_i & K_i \end{bmatrix} \begin{bmatrix} \bar{N}_i & 0_{(p_i - m_i) \times m_i} \\ 0_{m_i \times (n_i - m_i)} & I_{m_i \times m_i} \end{bmatrix} z_i \end{aligned}$$

in which  $\bar{N}_i = \begin{bmatrix} 0_{(p_i - m_i) \times (n_i - p_i)} & I_{(p_i - m_i) \times (p_i - m_i)} \end{bmatrix}$

In the sliding mode which can be further rewritten as

$$\begin{aligned} \sigma_i &= \begin{bmatrix} -N_i \bar{N}_i & K_i \end{bmatrix} \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} \\ &= -N_i \bar{N}_i z_{i1} + K_i z_{i2} = 0 \end{aligned} \quad (7)$$

For simplicity's sake, we select  $N_i = 0_{m_i \times (p_i - m_i)}$  so  $z_{i2} = 0$  and the sliding motion equation (4) can be rewritten as

$$\dot{z}_{i1} = (A_{ii1} + D_{ii1} F_{ii} E_{ii1}) z_{i1} + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ij1} z_{j1} \quad (8)$$

**Remark 1:** In general we can choose any matrix  $N_i \in R^{m_i \times (p_i - m_i)}$  but we select  $N_i = 0_{m_i \times (p_i - m_i)}$  for benefit in designing adaptive controller which is simple and very effective.

**Remark 2:** It is worthy to point out that the sliding function in (2) do not need the availability of the state variables that is very useful and can be applied for complex large scale system with no enough sensor for measuring system states.

### 3.2 Asymptotically stable conditions by LMI theory

Let us first consider the problem of sliding-surface design, the result of which is given in the form of LMI. The first result of designing sliding surface can be stated as follows.

**Theorem 1:** Suppose that the LMI (9)

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$$\begin{bmatrix} A_{ii}^T P_{ii} + P_{ii} A_{ii} + I + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ji}^T P_{jj} P_{jj} A_{ji} & D_{ii}^T P_{ii} & E_{ii}^T \\ P_{ii} D_{ii} & -I & 0 \\ E_{ii} & 0 & -I \end{bmatrix} \quad (9)$$

has solution  $P_{ii} > 0$ , sliding-surface is given by equation (2) and the resulting  $(n_i - m_i)$  reduced - order dynamics of the closed loop subsystem (8) restricted to the switching surface  $\sigma_i = 0$  is asymptotically stable.

Before proof the system (1) is asymptotically stable, we recall the following lemmas which will be needed in the inference of our results.

**Lemma 1** [31]: Let  $X$ ,  $Y$  and  $F$  be real matrices of suitable dimension with  $F^T F \leq I$  then, for any scalar  $\varphi > 0$ , the following matrix inequality holds:

$$XFY + Y^T F^T X^T \leq \varphi^{-1} X X^T + \varphi Y^T Y$$

**Lemma 2** [9]: Let  $X$ ,  $Y$  be real matrices of suitable dimension then, for any scalar  $\mu > 0$ , the following matrix inequality holds:

$$XY^T + YX^T \leq \mu X X^T + \mu^{-1} Y^T Y$$

**Lemma 3** [1]: The following matrix inequality:

$$\begin{bmatrix} Q(x) & \Pi(x) \\ \Pi(x)^T & R(x) \end{bmatrix} > 0$$

where  $Q(x) = Q(x)^T$ ,  $R(x) = R(x)^T$  and  $\Pi(x)$  depend affinity on  $x$ , is equivalent to  $R(x) > 0$ ,

$$Q(x) - \Pi(x) R(x)^{-1} \Pi(x)^T > 0$$

**Proof:** To analyze the stability of the sliding motion (8), we consider the following candidate of Lyapunov function:

$$V = \sum_{i=1}^L z_{i1}^T P_{ii} z_{i1} \quad (10)$$

Then, taking the time derivative along the state trajectory of system (10), we have

$$\begin{aligned} \dot{V} = & \sum_{i=1}^L z_{i1}^T \{ (A_{ii1} + D_{ii1} F_{ii} E_{ii1})^T P_{ii} + P_{ii} (A_{ii1} + D_{ii1} F_{ii} E_{ii1}) \} z_{i1} \\ & + \sum_{i=1}^L \sum_{\substack{j=1 \\ j \neq i}}^L (z_{i1}^T P_{ii} A_{ji1} z_{j1} + z_{j1}^T A_{ji1}^T P_{ii} z_{i1}) \end{aligned} \quad (11)$$

Then, by using Lemma 1, we have

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^L z_{i1}^T (A_{ii}^T P_{ii} + P_{ii} A_{ii1} + E_{ii1}^T E_{ii1} + P_{ii} D_{ii1} D_{ii1}^T P_{ii}) z_{i1} \\ & + \sum_{i=1}^L \sum_{\substack{j=1 \\ j \neq i}}^L (z_{i1}^T P_{ii} A_{ji1} z_{j1} + z_{j1}^T A_{ji1}^T P_{ii} z_{i1}) \end{aligned} \quad (12)$$

Then, using Lemma 2 yields that

$$\begin{aligned} \dot{V} \leq & \sum_{i=1}^L z_{i1}^T (A_{ii}^T P_{ii} + P_{ii} A_{ii1} + E_{ii1}^T E_{ii1} + P_{ii} D_{ii1} D_{ii1}^T P_{ii}) z_{i1} \\ & + \sum_{i=1}^L \sum_{\substack{j=1 \\ j \neq i}}^L (z_{i1}^T z_{j1} + z_{j1}^T A_{ji1}^T P_{ii} P_{ii} A_{ji1} z_{j1}) \\ = & \sum_{i=1}^L z_{i1}^T (A_{ii}^T P_{ii} + P_{ii} A_{ii1} + E_{ii1}^T E_{ii1} + P_{ii} D_{ii1} D_{ii1}^T P_{ii}) z_{i1} \\ & + \sum_{i=1}^L \sum_{\substack{j=1 \\ j \neq i}}^L (z_{i1}^T z_{j1} + z_{i1}^T A_{ji1}^T P_{jj} P_{jj} A_{ji1} z_{j1}) \\ = & \sum_{i=1}^L z_{i1}^T (A_{ii}^T P_{ii} + P_{ii} A_{ii1} + E_{ii1}^T E_{ii1} + P_{ii} D_{ii1} D_{ii1}^T P_{ii} \\ & + I + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ji1}^T P_{jj} P_{jj} A_{ji1}) z_{i1} \end{aligned} \quad (13)$$

From Lemma 3, by the Schur complement, equation (9) is equivalent to equation (14) and (15)

$$\begin{bmatrix} A_{ii}^T P_{ii} + P_{ii} A_{ii1} + I + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ji1}^T P_{jj} P_{jj} A_{ji1} & D_{ii}^T P_{ii} \\ P_{ii} D_{ii1} & -I \end{bmatrix} \quad (14)$$

$$+ \begin{bmatrix} E_{ii1}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{ii1} & 0 \end{bmatrix} < 0$$

$$A_{ii}^T P_{ii} + P_{ii} A_{ii} + E_{ii}^T E_{ii} + P_{ii} D_{ii} D_{ii}^T P_{ii} + I + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ji}^T P_{jj} P_{jj} A_{ji} < 0 \quad (15)$$

Thus, by equation (13) and (15), we have

$$\dot{V} \leq \sum_{i=1}^L z_{i1}^T (A_{ii}^T P_{ii} + P_{ii} A_{ii} + E_{ii}^T E_{ii} + P_{ii} D_{ii} D_{ii}^T P_{ii} + I + \sum_{\substack{j=1 \\ j \neq i}}^L A_{ji}^T P_{jj} P_{jj} A_{ji}) z_{i1} < 0 \quad (16)$$

Equation (16) shows that if LMI (9) holds, which further implies that sliding motion is asymptotically stable.

**Remark 3:** It is noted that the LMI condition in (9) is very simple and easy to satisfy compare with LMI in [12, 14].

### 3.3 Decentralized adaptive output feedback control law design

Now, the modified variable structure controller is selected to be

$$u_i = -(K_i B_{i2})^{-1} \{ \zeta_i + [\|K_i\|(\|A_{ii4}\| + \|D_{ii2}\| \|E_{ii2}\|) + \sum_{\substack{j=1 \\ j \neq i}}^L \|K_j\| \|A_{ji4}\|] \|K_i^{-1}\| \|\sigma_i\| + \alpha_i \|\sigma_i\| + k_{m_i} \|K_i\| \|B_{i2}\| \|H_{i2}\| \|K_i^{-1}\| \|\sigma_i\| \} \frac{\sigma_i}{\|\sigma_i\|} \quad (17)$$

And adaptive law is defined as:

$$\dot{\zeta}_i \geq \{ \|K_i\|(\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) + \sum_{\substack{j=1 \\ j \neq i}}^L \|K_j\| \|A_{ji3}\| \} \hat{\psi}_i + \frac{\eta_i^2}{4} \|\sigma_i\| \quad (18)$$

where  $\hat{\psi}_i$  is the solution of equation (19)

$$\dot{\hat{\psi}}_i = -\hat{\psi}_i \|\sigma_i\| + \|K_i\|(\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) + \sum_{\substack{j=1 \\ j \neq i}}^L \|K_j\| \|A_{ji3}\| \quad (19)$$

and

$$\hat{\psi}_i(t) = \eta_i - \tilde{\psi}_i(t) \quad (20)$$

**Theorem 2:** Suppose that the LMI (9) has solution  $P_{ii} > 0$ . Consider the closed loop of the mismatched uncertain large scale systems (1) with the above decentralized adaptive output feedback controller (17) where the linear sliding-function is given by equation (2). Then, the state trajectories of system (1) stay on the sliding surface thereafter and the stability of the overall system is also achieved for  $\|z_{i1}\| \leq \eta_i$ .

**Proof.** Since  $x_i = H_{i1} z_{i1} + H_{i2} z_{i2}$ , where  $T_i^{-1} = [H_{i1} \ H_{i2}]$ , so we have

$$\|x_i\| \leq \|H_{i1}\| \eta_i + \|H_{i2}\| \|K_i^{-1}\| \|\sigma_i\| \quad (21)$$

We consider the following positive definite function

$$V = \sum_{i=1}^L (\|\sigma_i\| + 0.5 \tilde{\psi}_i^2) \quad (22)$$

Then, by differentiating equation (22) along the trajectories of  $z_{i2}$ , we obtain

$$\dot{V} = \sum_{i=1}^L (\frac{\sigma_i^T}{\|\sigma_i\|} \dot{\sigma}_i + \tilde{\psi}_i \dot{\tilde{\psi}}_i) = \sum_{i=1}^L (\frac{\sigma_i^T}{\|\sigma_i\|} K_i \dot{z}_{i2} - \tilde{\psi}_i \dot{\tilde{\psi}}_i) \quad (23)$$

Thus, substituting equation (5) into the equation (23), we have

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$$\begin{aligned}
\dot{V} &= \sum_{i=1}^L \frac{\sigma_i^T}{\|\sigma_i\|} \{K_i(A_{ii3} + D_{ii2}F_{ii}E_{ii1})z_{i1} \\
&\quad + K_i(A_{ii4} + D_{ii2}F_{ii}E_{ii2})z_{i2} + K_iB_{i2}(u_i + \xi_i(x_i, t)) \\
&\quad + \sum_{j=1, j \neq i}^L K_i(A_{ij3}z_{j1} + A_{ij4}z_{j2})\} - \sum_{i=1}^L (\tilde{\psi}_i \dot{\psi}_i) \\
&= \sum_{i=1}^L \frac{\sigma_i^T}{\|\sigma_i\|} K_i \{ (A_{ii3} + D_{ii2}F_{ii}E_{ii1})z_{i1} + (A_{ii4} \\
&\quad + D_{ii2}F_{ii}E_{ii2})z_{i2} \} + \sum_{i=1}^L \frac{\sigma_i^T}{\|\sigma_i\|} K_i B_{i2} (u_i + \xi_i(x_i, t)) \\
&\quad + \sum_{i=1}^L \sum_{j=1, j \neq i}^L \frac{\sigma_i^T}{\|\sigma_i\|} K_i (A_{ij3}z_{j1} + A_{ij4}z_{j2}) - \sum_{i=1}^L (\tilde{\psi}_i \dot{\psi}_i) \\
&\leq \sum_{i=1}^L \|K_i\| \{ (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\|) \eta_i \\
&\quad + (\|A_{ii4}\| + \|D_{ii2}\| \|E_{ii2}\|) \|K_i^{-1}\| \|\sigma_i\| \} - \sum_{i=1}^L (\tilde{\psi}_i \dot{\psi}_i) \\
&\quad + \sum_{i=1}^L \sum_{j=1, j \neq i}^L \|K_j\| (\|A_{ji3}\| \eta_j + \|A_{ji4}\| \|K_i^{-1}\| \|\sigma_i\|) \\
&\quad + \sum_{i=1}^L \|K_i\| \|B_{i2}\| \|\xi_i(x_i, t)\| + \sum_{i=1}^L \frac{\sigma_i^T}{\|\sigma_i\|} K_i B_{i2} u_i \\
\dot{V} &\leq \sum_{i=1}^L \{ \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\| \} \eta_i \\
&\quad + \sum_{i=1}^L \{ \|K_i\| (\|A_{ii4}\| + \|D_{ii2}\| \|E_{ii2}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji4}\| \|K_i^{-1}\| \|\sigma_i\| \\
&\quad + k_{m_i} \|K_i\| \|B_{i2}\| \|H_{i2}\| \|K_i^{-1}\| \|\sigma_i\| \} \frac{\sigma_i}{\|\sigma_i\|} \\
&\leq \sum_{i=1}^L \{ \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\| \} \eta_i - \sum_{i=1}^L \zeta_i - \sum_{i=1}^L \alpha_i \|\sigma_i\| - \sum_{i=1}^L (\tilde{\psi}_i \dot{\psi}_i) \quad (25)
\end{aligned}$$

Since  $\|\xi_i(x_i, t)\| \leq k_{m_i} \|x_i\|$ , so we have

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^L \|K_i\| \{ (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\|) \eta_i \\
&\quad + (\|A_{ii4}\| + \|D_{ii2}\| \|E_{ii2}\|) \|K_i^{-1}\| \|\sigma_i\| \} - \sum_{i=1}^L (\tilde{\psi}_i \dot{\psi}_i) \\
&\quad + \sum_{i=1}^L \sum_{j=1, j \neq i}^L \|K_j\| (\|A_{ji3}\| \eta_j + \|A_{ji4}\| \|K_i^{-1}\| \|\sigma_i\|) \\
&\quad + \sum_{i=1}^L k_{m_i} \|K_i\| \|B_{i2}\| (\|H_{i1}\| \eta_i + \|H_{i2}\| \|K_i^{-1}\| \|\sigma_i\|) \\
&\quad + \sum_{i=1}^L \frac{\sigma_i^T}{\|\sigma_i\|} K_i B_{i2} u_i \quad (24)
\end{aligned}$$

From equation (24) and control law (17), we have

Thus, substituting equation (19) and equation (20) into equation (25), we obtain

$$\begin{aligned}
\dot{V} &\leq \sum_{i=1}^L \{ \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\| \} \eta_i - \sum_{i=1}^L \zeta_i - \sum_{i=1}^L \alpha_i \|\sigma_i\| \\
&\quad - \sum_{i=1}^L \tilde{\psi}_i [-\dot{\psi}_i \|\sigma_i\| + \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| \\
&\quad + k_{m_i} \|B_{i2}\| \|H_{i1}\|) + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\|] \\
&= \sum_{i=1}^L \{ \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\| \} \eta_i - \sum_{i=1}^L \zeta_i - \sum_{i=1}^L \alpha_i \|\sigma_i\| \\
&\quad - \sum_{i=1}^L \tilde{\psi}_i \{ \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\| \} + \sum_{i=1}^L \tilde{\psi}_i \dot{\psi}_i \|\sigma_i\| \\
&= \sum_{i=1}^L \hat{\psi}_i \{ \|K_i\| (\|A_{ii3}\| + \|D_{ii2}\| \|E_{ii1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{j=1, j \neq i}^L \|K_j\| \|A_{ji3}\| \} - \sum_{i=1}^L \zeta_i + \sum_{i=1}^L \tilde{\psi}_i \dot{\psi}_i \|\sigma_i\| - \sum_{i=1}^L \alpha_i \|\sigma_i\| \quad (26)
\end{aligned}$$

Then, using equation (18) and equation (26), we have

$$\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^L \{ (\|K_i\| (\|A_{i3}\| + \|D_{i2}\| \|E_{i1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{\substack{j=1 \\ j \neq i}}^L \|K_j\| \|A_{j3}\|) \|\hat{\psi}_i + \frac{\eta_i^2}{4} \|\sigma_i\| \} - \sum_{i=1}^L \alpha_i \|\sigma_i\| \\
&\quad + \sum_{i=1}^L \hat{\psi}_i \{ \|K_i\| (\|A_{i3}\| + \|D_{i2}\| \|E_{i1}\| + k_{m_i} \|B_{i2}\| \|H_{i1}\|) \\
&\quad + \sum_{\substack{j=1 \\ j \neq i}}^L \|K_j\| \|A_{j3}\| \} + \sum_{i=1}^L \tilde{\psi}_i \hat{\psi}_i \|\sigma_i\| \\
&= -\sum_{i=1}^L \frac{\eta_i^2}{4} \|\sigma_i\| + \sum_{i=1}^L \tilde{\psi}_i \hat{\psi}_i \|\sigma_i\| - \sum_{i=1}^L \alpha_i \|\sigma_i\| \\
&= -\sum_{i=1}^L \|\sigma_i\| \frac{\eta_i^2}{4} + \sum_{i=1}^L \|\sigma_i\| (\eta_i \hat{\psi}_i - \tilde{\psi}_i \hat{\psi}_i) - \sum_{i=1}^L \alpha_i \|\sigma_i\| \\
&= -\sum_{i=1}^L \|\sigma_i\| \frac{\eta_i^2}{4} + \sum_{i=1}^L \|\sigma_i\| \{ -(\hat{\psi}_i - \frac{\eta_i}{2})^2 + \frac{\eta_i^2}{4} \} \\
&\quad - \sum_{i=1}^L \alpha_i \|\sigma_i\| \\
&= -\sum_{i=1}^L \|\sigma_i\| (\hat{\psi}_i - \frac{\eta_i}{2})^2 - \sum_{i=1}^L \alpha_i \|\sigma_i\|
\end{aligned}$$

So

$$\dot{V} \leq -\sum_{i=1}^L \alpha_i \|\sigma_i\| \quad (27)$$

Equation (27) shows that the proposed adaptive controller (17) guarantees that the system states stay on the sliding surface thereafter and are invariant with matched uncertainties.

**Remark 4:** The closed loop of the uncertain subsystem (1) with the decentralized adaptive output feedback controller (17) and the new sliding surface is designed in (2) such that at any initial value the system states are forced onto sliding surface stay on it thereafter and invariant with external disturbance.

**Remark 5:** Applying observer to design the sliding mode control using only output variables were subject of many recently researches [6, 12, 14, 19] and [22]. The approach is to use state observers to provide an estimate of the unmeasured states. As a result, in the sliding mode the systems are asymptotically stable and have good performance. However, the proposed methods in [14, 19] and [22] require the system have certain structure and the approaches in [6] and [12] can not apply for systems with mismatched uncertainty in state matrix. So, in this paper we had solved the limitations of the above papers.

**Remark 6:** Adaptive law in controller (17) is very useful that make control energy depend upon output such that total control energy is small compare with [18] and [20].

**Remark 7:** the upper bound of unknown parameters  $\rho_i$ ,  $\xi_{ij}$  in equation (27) in paper [18] and  $\xi_1$ ,  $\xi_2$  in equation (13) in paper [20]) feed directly into the proposed controller in papers [18] and [20]. In this case, the value of the upper bound of unknown parameters is usually selected to be large for asymptotic stabilization of the system. So control energy must be large to force the system states into the sliding surface. In this study, the upper bound of unknown parameter  $\eta_i$  is auto turning dependent on output signal so that the control energy is smaller than the above paper.

**Remark 8:** The chattering phenomenon is highly undesirable because it may excite high-frequency unmodelled plant dynamics. There are some approaches to reduce the chattering.

The first, the discontinuous function  $\frac{\sigma_i}{\|\sigma_i\|}$  in the control input (17) is replaced by a continuous approximation such as

$\frac{\sigma_i}{\|\sigma_i\| + \varepsilon_i}$  where  $\varepsilon_i$  is positive constant [5]. This method can not

guarantee asymptotic stability but ultimate boundedness of system trajectories to within a neighborhood of the origin depending on  $\varepsilon_i$ . The second, the discontinuous controller (17) is replaced by a continuous one with a low switching frequency [18]. This approach is not only eliminatory the chattering problem but also guarantees asymptotic stability.

**Remark 9:** The decentralized control problem of uncertain large scale systems has attracted a lot of attention. Decentralized control issues naturally arise from the control of large complex systems found in the power industry, aerospace and chemical engineering applications, telecommunication network, etc. This study can apply to control many practical systems such as a river pollution problem [28], communication satellites, solar panels [7] and flywheel energy storage systems.

## 4. Numerical Example

In this section, we present a numerical example to show the advantages of the control schemes proposed in this paper. Consider a large scale systems composed of two third-order



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subsystems [24].

The first subsystem's dynamics are given as

$$\begin{aligned} \dot{x}_1 &= (A_{11} + \Delta A_{11})x_1 + B_1(u_1 + \xi_1(x_1, t)) + A_{12}x_2 \\ y_1 &= C_1x_1 \end{aligned}$$

where  $A_{11} = \begin{bmatrix} -8 & 0 & 1 \\ 0 & -7 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ ,  $B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ ,  $C_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and  $A_{12} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ . It is easy to verify that the pairs

$A_{11}, B_1$  are stabilizable. So, the assumption 1 is satisfied. The rank of matrix  $B_1$  is 1 and the rank of matrix  $C_1$  is 2. So, the assumption 2 is satisfied. The disturbance is

$\|\xi_1(x_1, t)\| \leq k_{m_1} \|x_1\|$  with  $k_{m_1} = 1.5$ . The mismatched uncertain

is  $\Delta A_{11} = D_{11}F_{11}E_{11}$  with  $D_{11} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $E_{11} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$  and

$F_{11} = \mathcal{G}_1 \sin(t) = 0.3 \sin(0.5t)$ . Obviously, the mismatched term  $\|F_{11}\| \leq 1$  so the assumption 4 is also satisfied.

The second subsystem's dynamics are given as

$$\begin{aligned} \dot{x}_2 &= (A_{22} + D_{22}F_{22}E_{22})x_2 + B_2(u_2 + \xi_2(x_2, t)) + A_{21}x_1 \\ y_2 &= C_2x_2 \end{aligned}$$

where  $A_{22} = \begin{bmatrix} -6 & 0 & 1 \\ 0 & -7 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ ,  $B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ ,

$A_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$  and  $C_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ . The assumption

1 is satisfied because the pairs  $A_{22}, B_2$  are controllable. The rank of matrix  $B_2$  is 1 and the rank of matrix  $C_2$  is 2. So, the

assumption 2 is satisfied. The disturbance is

$\|\xi_2(x_2, t)\| \leq k_{m_2} \|x_2\|$  with  $k_{m_2} = 2$ . The mismatched

uncertain is  $\Delta A_{22} = D_{22}F_{22}E_{22}$  with  $D_{22} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ ,

$E_{22} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$  and  $F_{22} = \mathcal{G}_2 \sin(t) = 0.2 \sin(t)$ . The assumption 4 is also satisfied because of the mismatched term  $\|F_{22}\| \leq 1$ .

Since the rank( $C_1B_1$ ) and rank( $C_2B_2$ ) are 1, according to the algorithm given in [31], the coordinate transformation

$$\bar{z}_i = T_i x_i \quad \text{with} \quad T_1 = T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad \text{Hence, the}$$

corresponding sliding surface for subsystem 1 and subsystem 2 are  $\sigma_1 = \begin{bmatrix} 0 & 1 \end{bmatrix} y_1 = 0$  and  $\sigma_2 = \begin{bmatrix} 0 & 1 \end{bmatrix} y_2 = 0$ . Then, by solving the LMI (9), we have solution for positive matrix  $P_{ii}$

with  $P_{11} = \begin{bmatrix} 0.6912 & 0.4579 \\ -0.4579 & 0.7228 \end{bmatrix}$  and  $P_{22} = \begin{bmatrix} 0.3848 & 0.1442 \\ -0.1442 & 0.3725 \end{bmatrix}$ .

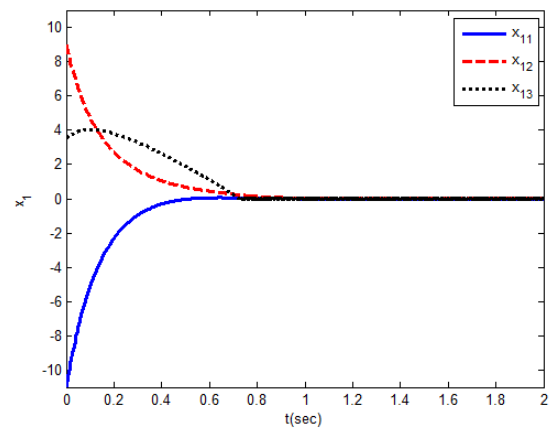
So, the Theorem 1 is held. Since  $\eta_1 = \eta_2 = 0.02$ , from Theorem 2 the controller for subsystem 1 and subsystem 2 are

$$u_1 = -(3.366\hat{\psi}_1 + 2.5\|\sigma_1\|) \frac{\sigma_1}{\|\sigma_1\|} \quad \text{and}$$

$$u_2 = -(3.8661\hat{\psi}_2 + 4.0001\|\sigma_2\|) \frac{\sigma_2}{\|\sigma_2\|} \quad \text{where } \hat{\psi}_1 \text{ and } \hat{\psi}_2 \text{ are}$$

solution of  $\dot{\hat{\psi}}_1 = -\hat{\psi}_1 \|\sigma_1\| + 3.366$  and

$$\dot{\hat{\psi}}_2 = -\hat{\psi}_2 \|\sigma_2\| + 3.8661.$$



**Fig.1. The time histories of  $x_{11}$ ,  $x_{12}$ ,  $x_{13}$  of subsystem 1**

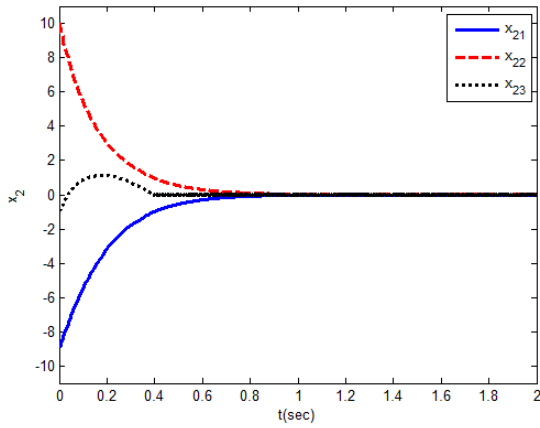


Fig.2. The time histories of  $x_{21}$ ,  $x_{22}$ ,  $x_{23}$  of subsystem 2

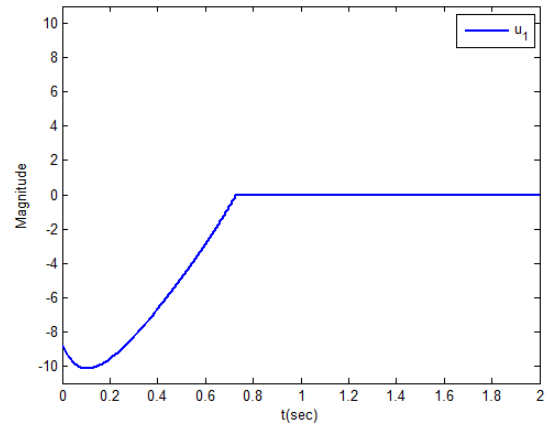


Fig.5. Time responses of the control input  $u_1$  of subsystem 1

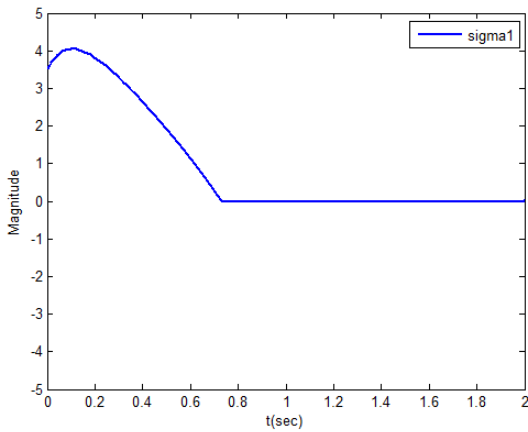


Fig.3. Time responses of the sliding function  $\sigma_1$  of subsystem 1

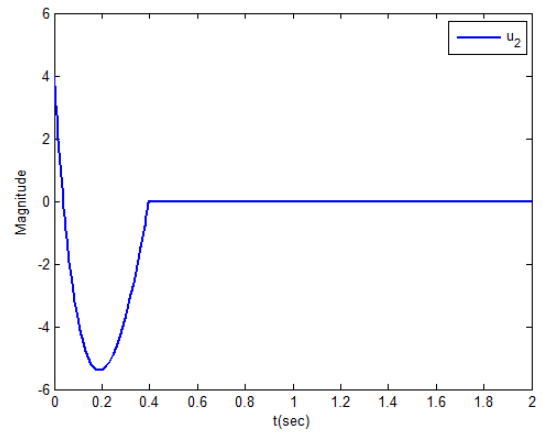


Fig.6. Time responses of the control input  $u_2$  of subsystem 2

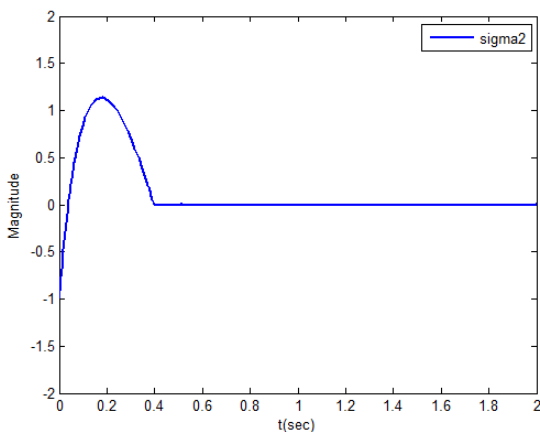


Fig.4. Time responses of the sliding function  $\sigma_2$  of subsystem 2

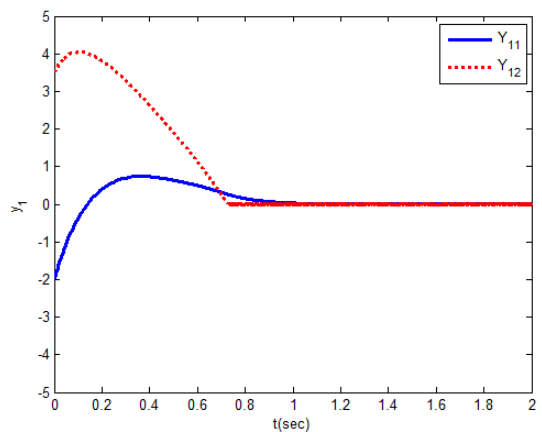


Fig.7. Time responses of the output  $y_1$  of subsystem 1

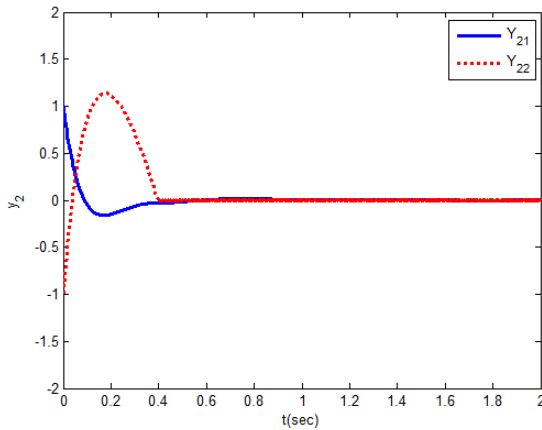


Fig.8. Time responses of the output  $y_2$  of subsystem 2

Simulation results for the first subsystem and the second subsystem are shown, respectively, in Fig. 1 to Fig. 8 with initial states  $x_1 = [-11; 9; 3.5]$  and  $x_2 = [-9; 10; -1]$ . It can be seen that the proposed method is effective in dealing with matched and mismatched uncertain and the system has a good performance.

**Remark 10:** It should be pointed out that the proposed controller uses only output variables even though the subsystem 1 and subsystem 2 have mismatched uncertainty in the state matrices and the disturbances are not the function of outputs. So, this approach is much more general than the proposed method in paper [24]. With the same initial states, from Fig 1 and Fig 2, we can see that the system performance by the present study is better than the proposed method in paper [24].

## 5. Conclusion

In this paper, decentralized adaptive output feedback stabilization of large-scale systems with mismatched uncertainties and exogenous disturbances is considered. Especially, this paper presents a solution to decentrally stabilize systems with sliding surface and adaptive controller using only output variables. Both the sliding surface and adaptive sliding-mode controller can be easily achieved via linear matrix inequality technique. Our approach does not need the availability of the state variables so that our method is very useful and more realistic since it can be easily implemented in practice. Finally, a numerical example is given to prove the

synthesis procedure for the proposed decentralized adaptive output feedback controller.

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