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An Optimal Maintenance Policy for the Leased System under Irregular Inspection

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ABSTRACT

A maintenance policy is proposed herein for a leased system under irregular inspection. Such policy is mainly at each inspection time to determine the maintenance action optimally and also to determine the next inspection time based on the current system conditions. Such policy holds by considering that: (1) each maintenance taken is regarded as a “minimal repair”; (2) the times spent on inspection and maintenance action are ignored. An example is presented to show how such policy can be carried out for such leased system in terms of the maintenance data generated through applying simulation. The simulated results illustrate that the total cost and the total operating cost of the system by taking such policy can reduce about 4% and 35% respectively.
***Key Words*:** maintenance policy, leased system, inspection, minimal repair

租賃系統不定時偵測下之最佳維護策略

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摘　要

本文為不定時偵測之租賃系統，提出一個最佳維護策略。該策略主要在每一個偵測時間，進行最佳動作之選定，並且依當時之系統狀態，選擇下一個偵測點。本文假設：1. 每一個選定之維護動作皆為 “最小維護”；2. 偵測時間與維護時間皆可忽略。最後；利用模擬說明該租賃系統如何進行本維護策略。由模擬結果，我們得到該系統採用此策略後，總成本與總操作成本可分別減少 4% 與 35%。
關鍵詞：維護策略，租賃系統，偵測，最小維護

I. INTRODUCTION

Since 1990, there has been a trend towards leasing equipment rather than owning the equipment. Due to usage and/or ageing, the leased system deteriorates worse and ultimately fails [1]. Such deterioration incurs the system failures and the associated penalty costs. As a result, maintenance was no longer an issue for the lessee who leases the equipment, but for the lessor (i.e. the owner of the equipment) who carried out the maintenance. Often, the contract specifies the penalties while the leased equipment not perform as required (for example, failing frequently) or the maintenance service not being satisfactory (for example, repairs are not performed within reasonable times limits) as these affect the business performance of the lessee.

Two kinds of maintenances are always taken into consideration, through corrective maintenance (CM), the failure system can be restored to an operational one; and through preventive maintenance (PM) the rate of degradation can be controlled. While the level of maintenance effort increases, the penalty costs decrease but the of maintenance costs increase. This implies that the optimal maintenance actions need to be decided by taking the penalty costs into consideration. The references deal with the sequential PM policies are Nakagawa [8] and Sheu and Change [10]. Those deal with periodic PM policies are Canfield [2], Dohi, et al. [3], Nakagawa [7] and Park, et al. [9]. Wang [10] deals with a survey of the maintenance policies for the deteriorating systems. Kyung [5] considered a continuous deterioration system which fails when its deterioration level surpasses a breakdown threshold. Under periodic inspections, such policy is to determine the optimal threshold γ^* to get the minimum long-run total mean cost per unit time. Accordingly, the system is replaced if the current deterioration level exceeds such γ^*, and do nothing if otherwise. Dieulle, et al. [6] deals with a continuous deterioration system which is inspected at random times which is chosen by a scheduling function $m(t)$. Such policy is mainly to determine both the optimal scheduling function $m^*(t)$ and the optimal threshold state M^* to minimize the long-run expected cost per unit time. Two type replacements then will be taken upon the current state X_t according to that: 1) if $X_t \geq L$, "*corrective replacement*" is taken, where L is the pre-set failure level, 2) if $L > X_t \geq M^*$, "*preventive replacement*" is taken, 3) otherwise, "*do nothing*". Jaturonnatee, et al. [4] develops a PM policy for a leased system by considering that: 1) each maintenance action taken is "*minimal repair*", 2) the failures produced by the system with/without maintenances occur according to a non-homogeneous Poisson process (NHPP). Such policy is mainly to determine the optimal parameters $(k, \underline{t}, \underline{\partial})$ to minimize the expected total cost over the lease period, where k denotes the number of PM actions to be carried out over the lease period, $\underline{t} = (t_1, ..., t_k)$ the time instants for the PM actions, $\underline{\partial} = (\partial_1, ..., \partial_k)$ the restored level of the PM actions.

In this article, we focus our attention on the leased system which is originally presented in [4]. A maintenance policy is proposed herein for such system to determine at each inspection time both the optimal maintenance action and the next inspection time based on the current system conditions. Section II describes such leased system and policy assumptions. Next, Section III presents the maintenance model in detail for deriving expected total cost over the planning period. Subsequently, an example is presented in Section IV to illustrate how such policy can be carried out for such system through simulation. Finally, some conclusions are drawn.

II. SYSTEM DESCRIPTIONS AND POLICY ASSUMPTIONS

The leased system considered herein satisfies that:

1. The deterioration of such system is time-dependent.
2. The leased period is L.
3. The contract involves the penalty costs for the lessor while the system failures occur during the leased period. Under no maintenance case, the associated penalty cost occurs over the leased period $[0, L]$ is given by $P_{(0,L)} = C^p \cdot N_L^0$.
4. The failures produced by such system with\without maintenances occur both according to the non-homogeneous Poisson process (NHPP). Accordingly, we let $\Lambda(t)$ and $\lambda_0(t) \forall t \geq 0$ denote the failure intensity functions of such system with and without maintenances respectively. In particular, such $\lambda_0(t)$ is a non-decreasing function to reflect the time-dependent deterioration.

To carry out the maintenance policy for the leased system, more assumptions are further required as followings:

A1: At each T_n for $T_n < L$, one action chosen from A={a^I, a^r, a^R} will be taken. While $T_n \geq L$, the system is replaced and a new leased period begins.

A2: While a^r is taken, the hazard function immediately after such action being taken is the same as that just before failure [3].

A3: The times spent on inspection and maintenance are both neglected.

A4: State F which denotes the system's sudden and temporary interruption due to fatal shock may also occur. The occurrence of F is self-indicative but the probability that such F occurs at each T_n is ignored. Whenever F occurs, the minimal repair is taken immediately in negligible time and costless.

III. THE MAINTENANCE POLICY

At T_n, the maintenance decision for the leased system is made according to the following steps:

Step 1. Determine the optimal maintenance action A_n^*.

Step 2. Determine the next inspection time T_{n+1}.

Step 3. Take A_n^* immediately.

How to determine A_n^* and T_{n+1} at T_n are illustrated in the following sections.

1. The Sscheduling Inspection Function $m(\cdot)$

Due to that the system deterioration is time-dependent; such system should be inspected more frequently when time goes on to identify the current system conditions in time. For this purpose, it is supposed that such system is inspected at T_n and at it the maintenance decision is made. We allow irregular inspection date in this study. Thus, the next inspection time T_{n+1} will be dynamically updated on the basis of the present system condition revealed by inspection to reflect the deterioration property.

Let $m(\cdot)$ be a decreasing function from $\left[0, N_L^A\right)$ to $\left[m_{\min}, m_{Max}\right]$. At T_n, such $m(\cdot)$ is noted by

$$m\left(N_{T_n}^A\right) = m_{\min} + Max\left(\left\lfloor (m_{Max} - m_{\min}) - \frac{(m_{Max} - m_{\min}) \cdot N_{T_n}^A}{K} \right\rfloor, 0\right) \tag{1}$$

where

A. m_{Max} and $m_{\min}$ denote the first inspection time (i.e. $m_{Max}=T_1$) and the minimal inspection period respectively.

B. $\lfloor x \rfloor = Max\{k \text{ is int eger} \mid k \le x\}$.

C. $N_{T_n}^A = N_{T_n}^A + \Delta_{(T_{n-1},T_n)}(a^k)$ in which $\Delta_{(T_{n-1},T_n)}(a^k)$ can be identified only by inspection at T_n in neglected time.

D. K is a pre-set constant number.

Consequentially, the sequence of inspection times $(T_n)_{n=0,1,2,\ldots}$ is strictly increasing and the possibility of an infinite number of inspections occurring on a finite interval is avoided. We then choose T_{n+1} by

$$T_{n+1} = T_n + m\left(N_{T_n}^A\right) \tag{2}$$

In case that we get $N_{T_n}^A = \sigma_n$ at the inspection time $T_n = t_n$, then such T_{n+1} will be obtained by $T_{n+1} = t_n + m(\sigma_n)$.

2. The Determination of A_n^*

The associated penalty costs occur during $[T_n, T_{n+1}]$ will increase while time goes on due to that the system deterioration is time-dependent. We should take proper maintenance to reduce the number of system failures during the leased period. This decreases the associated penalty costs but increase the expense of maintenance costs. It implies that the optimal maintenance action taken at T_n needs to be decided through a proper trade-off between penalty and maintenance costs.

Now, we show how to determine the optimal maintenance action A_n^* at T_n as followings: First of all, we need to determine at T_n the failure intensity function $\Lambda(t)$ given that the maintenance actions have been taken at $T_s=t_s$ for $s=1,\ldots, n-1$. Such $\Lambda(t)$ can be determined by

$$\Lambda(t) = \begin{cases} \lambda_0(t) & \text{for } 0 \le t < t_1 \\ \lambda_1(t) = \lambda_0(t) - \delta_1^k & \text{for } t_1 \le t < t_2 \\ \ldots & \\ \lambda_{n-1}(t) = \lambda_{n-2}(t) - \delta_{n-1}^k & \text{for } t_{n-2} \le t < t_{n-1} \\ \lambda_n(t) = \lambda_{n-1}(t) - \delta_n^k & \text{for } T_n \le t \end{cases} \tag{3}$$

where

A. $\delta_s^k = \theta^k \cdot \lambda_{s-1}(t_s)$, which denotes the reduction in the intensity function restored by a^k taken at $T_s = t_s$.

B. $\lambda_{s-1}(t_s)$ for $t_{s-1} \le t < t_s$ denotes the intensity function just before a^k being taken at $T_s = t_s$. Fig. 1 illustrates the failure intensity functions of the systems with/without maintenances.

Suppose that the candidate action a^k is taken at $T_n=t_n$, then the expected number of failures produced over the planning period $\left[t_n, t_n + M\right]$ can be determined by

$$\begin{aligned} &E(\Delta_{(t_n,t_n+M)}(a^k)) \\ &= \int_{t_n}^{t_n+M} \lambda_n(t)dt \\ &= \int_{t_n}^{t_n+M} \left(\lambda_{n-1}(t) - \theta^k \cdot \lambda_{n-1}(t_n)\right)dt \quad \text{for } k = I, r \end{aligned} \tag{4}$$

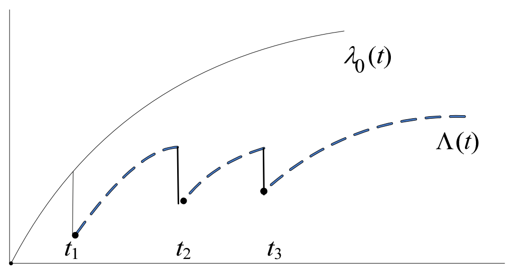

Fig. 1. The failure intensity functions of the systems with/without maintenances

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where M is a pre-set integer for decision use. While a^R is taken, a new leased period begins.

Let $\tau_{(T_n,T_n+M)}\left(a^k\right)$ be the associated total expected cost spent over [T_n, T_n+M] while a^k is taken at T_n. It is noted that

$$\tau_{(T_n,T_n+M)}(a^k) = \begin{cases} \tau_{(T_n,T_n+M)}(a^I) = C^I + E\left(P_{(T_n,T_n+M)}\left(a^I\right)\right) & \text{if } a^I \text{ is taken} \\ \tau_{(T_n,T_n+M)}(a^r) = C^I + C\left(a^r\right) + E\left(P_{(T_n,T_n+M)}\left(a^r\right)\right) & \text{if } a^r \text{ is taken} \\ \tau_{(T_n,T_n+M)}(a^R) = C^I + C\left(a^R\right) & \text{if } a^R \text{ is taken} \end{cases} \quad (5)$$

where

A. C^I and $C\left(a^k\right)$ denote the cost to take each inspection and a^k respectively.

B. $E\left(P_{(T_n,T_n+M)}\left(a^k\right)\right) = C^p \cdot E(\Delta_{(T_n,T_n+M)}(a^k))$ in which $P_{(T_n,T_n+M)}\left(a^k\right)$ denotes the associated penalty cost occurs over [T_n, T_n+M] in case that a^k is taken at T_n.

We thus choose the optimal maintenance action A_n^* at T_n by

$$A_n^* = \arg \min_{k\in\{I,r,R\}} \left\{\tau_{(T_n,T_n+M)}\left(a^k\right)\right\} \quad (6)$$

3. The Maintenance Procedure

For convenience, a procedure named as "$D\left(A_n^*, T_{n+1} \mid M\right)$" is proposed herein to illustrate how the determinations of A_n^* and T_{n+1} can be carried out at T_n based on the current system conditions in the following steps:

Step 0: Choice a proper integer M for decision use.

Step 1: Determine $\tau_{(T_n,T_n+M)}\left(a^k\right)$ by Eq.(5).

Step 2: Determine A_n^* by Eq.(6).

Step 3: Determine T_{n+1} according to the following sub-steps:

3-1. Identify $\Delta_{(T_{n-1},T_n)}(a^k)$ by inspection.

3-2. Determine the total number of failures at T_n (say $N_{T_n}^A = \sigma_n$).

3-3. Determine $m(\sigma_n)$ by Eq.(1).

3-4. Determine T_{n+1} by Eq.(2). #

IV. ILLUSTRATION

A leased system under inspection at each T_n is presented herein to illustrate how the proposed maintenance policy can be carried out for it. Let $\lambda_0(t) = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}$ be the hazard function of such leased system where α=100, β=100 to indicate an increasing failure rate. For our purpose, other numerical settings are further required as followings: $m_{\min}$=1, m_{Max}=10, K=500, L=100(day), θ^r=0.4, θ^I=0, M=15(day), C^I=5($), C^p=30($), $C(a^r)$ =500($), and $C(a^R)$ =100000($).

1. The Maintenance History of the System under the Policy

Simulation is applied to generate the maintenance data at each T_n, and then at it both A_n^* and T_{n+1} are determined respectively till the first integer T that $T > L$. Such simulation is carried out according to the followings:

A. Firstly, we set $T_0 = N_0^A = 0$ and $T_1 = m_{Max}$.

B. At each T_n for $T_n < L$:

B-1. Determine A_n^* and T_{n+1} by $D\left(A_n^*, T_{n+1} \mid M = 15\right)$.

B-2. Determine the resulting intensity function $\lambda_n(t)$ for $t \geq T_n$ immediately after A_n^* is taken at T_n according to:

- If $A_n^* = a^I$, let $\lambda_n(t) = \lambda_{n-1}(t)$ for $t \geq T_n$.
- If $A_n^* = a^r$, let $\lambda_n(t) = \left(\lambda_{n-1}(t) - \theta^r \cdot \lambda_{n-1}(T_n)\right)$ for $t \geq T_n$.

C. The procedure stops either $A_n^* = a^R$ or $T_n \geq L$.

The results are summarized in Table 1. The benefits of such system by taking maintenances are obtained through comparing such results to that under no maintenances in terms of two parameters $R_{top}(C^p, m_{Max}, M)$ and $R_{tc}(C^p, m_{Max}, M)$ respectively, where

$$R_{top}(C^p, m_{Max}, M) = \frac{\text{Total operating cost for the system with maintenance given}(C^p, m_{Max}, M)}{\text{Total operating cost for the system without maintenance}} \quad (7)$$

$$R_{tc}(C^p, m_{Max}, M) = \frac{\text{Total cost for the system with maintenance given}(C^p, m_{Max}, M)}{\text{Total cost for the system without maintenance}} \quad (8)$$

The total operating cost discussed herein includes penalty cost, maintenance cost and inspection cost. The total cost is the sum of total operating cost and replacement cost. Simulation is also applied for the system under no maintenance to generate the inspected data at each T_n till replacement is taken. We have that R_{tc}(30,10,15)=0.960 and R_{top}(30,10,15)=0.667 in Table 1. That is, the leased system if taking the maintenance policy will save 4% in total cost and save 33.3% in total operating cost. We further evaluate such R_{top}(30, m_{Max}, M) and R_{tc}(30, m_{Max}, M) for more $M\in\{12,13,17,20,25,30\}$ and $m_{Max}\in\{8,11,15,18,20\}$.

Table 1. The results of the system under the proposed policy

Policy	Maintenance cost	Inspection cost	Penalty cost	$R_{tc}(30,10,15)$	$R_{top}(30,10,15)$	Maintenance time
Without maintenance	-	85	13475.66	-	-	NA
With maintenance	2000	65	6971.55	0.960	0.667	38, 54,69, 89

The results are illustrated in Fig. 2. From Fig. 2, we observe that: 1) while (C^p, m_{Max})=(30, 10), if we choose M=20, the system by taking the policy can save 40.3% in total operating cost, 2) while (C^p, M)=(30, 15), if we choose m_{Max}=15, the system by taking the policy can save 38.5% in total operating cost.

2. Sensitivity Studies

The effects of various parameters on the optimal policy are also studied herein by considering more parameter values as indicated that: A. $C^p \in \{20,50\}$; B. $M \in \{18,10\}$; and C. $m_{Max} \in \{5,8,9,11,18,20\}$. The benefits for each considered combination are summarized in Table 2 and Figure 4-6 in terms of $R_{top}(C^p, m_{Max}, M)$ and $R_{tc}(C^p, m_{Max}, M)$ respectively. Some observations from Table 2 and Figure 4-6 are summarized as follows:

A. The effect of parameter m_{Max} on the benefit is insignificant.

B. While C^p gets larger, the system by taking maintenances can get more benefits than the no maintenance case.

C. For $C^p \in \{20,30,50\}$, the system by taking maintenances can get more benefits than the no maintenance case while M gets larger.

D. In order to get more benefits, M should be chosen properly for different C^p. In this study, M=18 and 15 seem to be the better choices for C^p =30 and 50 respectively.

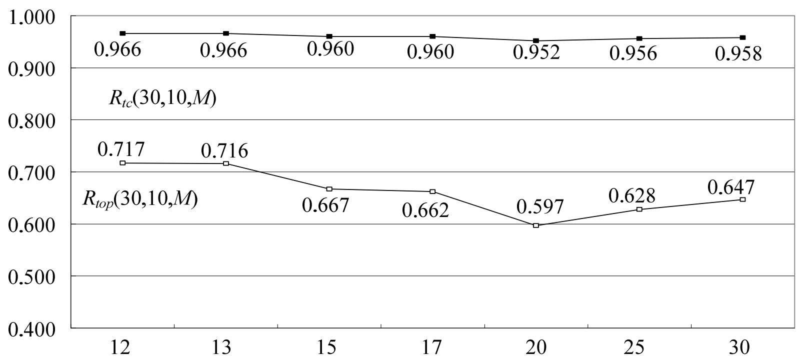

Fig. 2. The benefits of the system under maintenances for $M \in \{12,13,15,17,20,25,30\}$

Table 2. The benefits for the considered combinations

	M=10						M=15					
	C^p=20		C^p=30		C^p=50		C^p=20		C^p=30		C^p=50	
m_{Max}	$R_{tc}(\cdot)$	$R_{top}(\cdot)$	$R_{tc}(\cdot)$	$R_{top}(\cdot)$	$R_{tc}(\cdot)$	$R_{top}(\cdot)$	$R_{tc}(\cdot)$	$R_{top}(\cdot)$	$R_{tc}(\cdot)$	$R_{top}(\cdot)$	$R_{tc}(\cdot)$	$R_{top}(\cdot)$
5	0.997	0.996	0.977	0.800	0.927	0.585	0.986	0.820	0.962	0.672	0.917	0.529
8	0.996	0.949	0.973	0.772	0.924	0.578	0.986	0.757	0.961	0.667	0.913	0.518
10	0.997	0.959	0.973	0.772	0.923	0.583	0.986	0.760	0.960	0.667	0.906	0.487
11	0.997	0.945	0.975	0.792	0.928	0.610	0.986	0.827	0.960	0.664	0.907	0.500
15	0.997	0.965	0.976	0.766	0.920	0.576	0.979	0.752	0.952	0.615	0.910	0.527
18	0.995	0.944	0.970	0.762	0.920	0.586	0.986	0.836	0.958	0.666	0.905	0.514
20	0.997	0.962	0.971	0.778	0.923	0.611	0.991	0.897	0.954	0.646	0.907	0.527

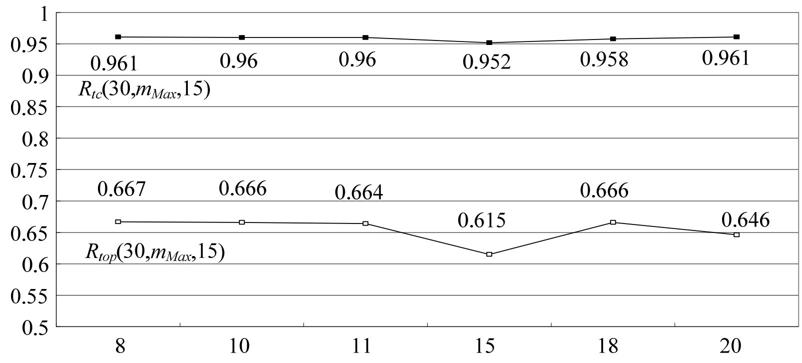

Fig. 3. The benefits of the system under maintenances for m_{Max}∈{8,10,11,15,18,20}

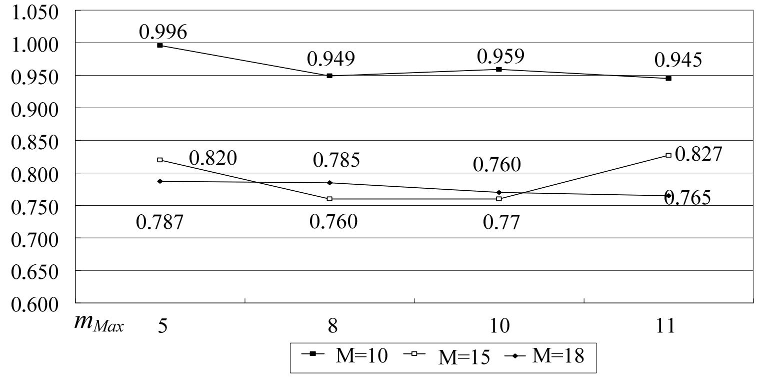

Fig. 4. The benefit R_{top}(*20*, m_{Max}, *M*) of the system under maintenance

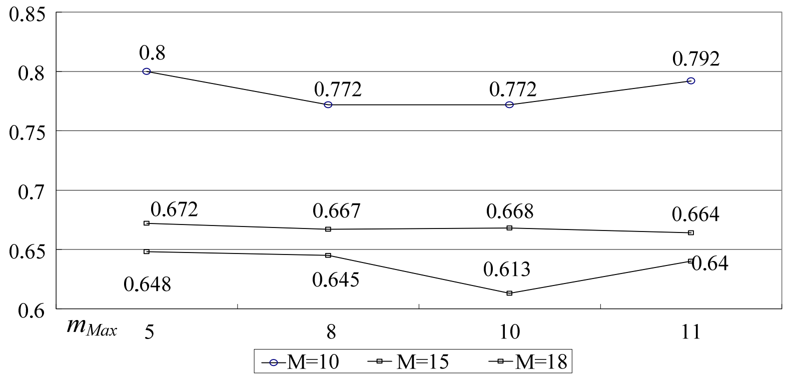

Fig. 5. The benefit R_{top}(30, m_{Max}, *M*) of the system under maintenance

V. CONCLUSION

A maintenance policy is proposed for the leased system which is originally presented in Jaturonnatee, et al [4]. Such system is under irregular inspection herein and the inspection date is dynamically updated based on the total number of failures identified to reflect the time-dependent system deterioration. Such policy is mainly to determine at each T_n the optimal maintenance action A_n^* and the next inspection time T_{n+1}. The policy can always get the minimum expected total cost over the leased periods by adjusting automatically both A_n^* and T_{n+1} based on the current system conditions. Further improvements may go on along the lines: 1. take different scheduling inspection function into consideration, say $m(\cdot)$ is a quadratic function, 2. take multi-action into consideration, and 3. determine the planning period M as a function of C^p.

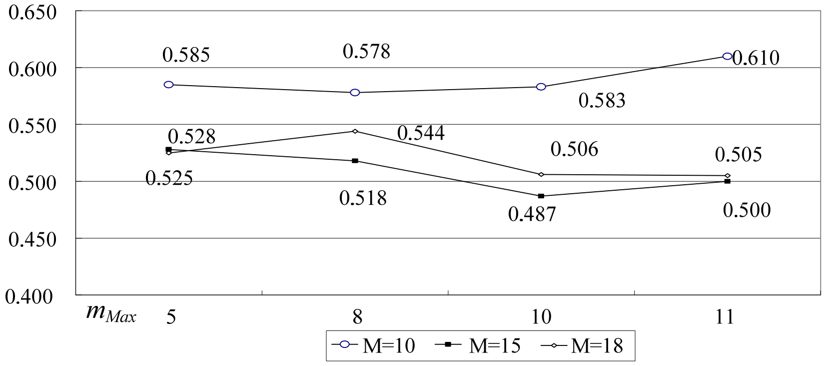

Fig. 6. The benefit $R_{top}(50, m_{Max}, M)$ of the system under maintenance

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NOMENCLATURE

T_n	The *n*th inspection time at which the *n*th maintenance decision is also made.
$A=\{a^I, a^r, a^R\}$	The candidate maintenance set at each T_n, where a^I, a^r and a^R denote "*Inspection*", "*repair*" and "*Replacement*" respectively.
C^p	The penalty cost due to per system failure.
$\Lambda(t)$ and $\lambda_0(t)$ $\forall(t)\geq 0$	The failure intensity functions of the system with and without maintenances respectively.
$m(\cdot)$	The scheduling inspection function.
N_L^0	The random number of failures produced by the system under no maintenances over the leased periods $[0,L]$.
N_L^A and $N_{T_n}^A$	The random numbers of failures produced by the system under maintenances over $[0,L]$ and $[0,T_n]$ respectively.
$\Delta_{(T_{n-1},T_n)}(a^k)$	The random number of failures produced over $[T_{n-1},T_n]$ in case that a^k is taken at T_{n-1}.
$P_{(0,L)}$	The penalty cost occurs over the leased period $[0,L]$ while the system is under no maintenances.
$P_{(T_n,T_n+M)}(a^k)$	The penalty cost occurs over $[T_n,T_n+M]$ while a^k is taken at T_n.
$\tau_{(T_n,T_n+M)}(a^k)$	The associated total expected cost spent over $[T_n,T_n+M]$ while a^k is taken at T_n.