Theoretical Analysis of an MC-CDMA System in Cellular Environments

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ABSTRACT
By extending the scheme adopted by Chen [13], the performance of an improved cellular MC-CDMA (multi-carrier coded-division multiple-access) system for operation within a local cell, plus 12 surrounding cells in the first two tiers, is evaluated in this report. Herein, the impact of system performance with different geometrical structures of the local and the neighboring cells (the six furthest cells) is illustrated in the results from the number simulation implemented. The evaluation of error probability for an MC-CDMA system is investigated from the viewpoint of theoretical analysis, in which a working environment with both single-cell and multiple-cell configurations and correlated-Nakagami-m fading channel statistics is assumed. A joint characteristic function is applied to determine the JPDF (joint probability density function), including a generalized Laguerre polynomial. For the sake of simplicity, the difficult traditional methods for explicitly obtaining the JPDF are avoided. In this study, some new closed-form formulas for the average BER (bit error rate), with statistical calculation of MAI (multiple access interference) for an MC-CDMA system with MRC (maximal ratio combining) diversity operating in a multiple-cell environment, are also obtained.

Key Words: MC-CDMA, MRC diversity, joint characteristic function, generalized laguerre polynomial, correlated-Nakagami-m fading

基於理論分析觀點研究蜂巢環境中之 MC-CDMA 系統

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摘 要
本篇論文對改良的蜂巢式多載波分碼多重近接 (multi-carrier coded-division multiple-access, MC-CDMA) 系統作系統效能之分析，其中利用擴展文獻 [13] 內容，針對 MC-CDMA 系統工作於區域之蜂巢中，有 12 個蜂巢周遭之環境進行效能分析。分析過程，介於本區蜂巢與相鄰蜂巢 (考慮 6 個較近的蜂巢) 不同幾何形狀進行說明於數據分析中。此一針對 MC-CDMA 系統之錯誤率的評估，係基於研究之理論分析為出發點；其中之系統工作環境假設單一與多重之蜂巢結構，而假設相關 Nakagami-m 之通道統計分布，傳統對於求取聯合機率密度函數 (joint probability density function, JPDF) 是有某些程度之困難，爲了避難並簡化，利用歸化一般化之 Laguerre 多項式的聯合特徵函數，以決定聯合機率密度函數，其中對於 MC-CDMA 系統結合最大比例合成 (maximal ratio combining, MRC) 分集工作於多蜂巢環境，並且計入多重近接干擾
Due to reflection, diffraction, and scatter exist in the wireless cellular communication systems, the propagation channel exhibits substantially multipath fading behavior. Whenever compensation for the losses in multipath fading is desired, the diversity-combining technique is one of the effective methods [22]. In earlier studies, the statistical models of a fading channel concentrated on the assumption that the channel diversity branches were statistically independent of each other; whereas, such consideration should be based on the assumption that the paths are sufficiently separated [18]. It is known that the impact of correlation on performance has been proved to exist both in single-cell and multiple-cell direct-sequence coded-division multiple-access communication systems (DS-CDMA) [3].

The multiple-access techniques with multi-carriers for multiple user environments, e.g., MC-CDMA (multi-carrier coded-division multiple-access) systems or the direct-sequence type, i.e., MC-DS-CDMA (multi-carrier direct-sequence coded-division multiple-access) systems, have become the standard techniques for achieving next-generation wireless cellular systems. In the past a large number of papers proposing the new architecture and evaluating the performance for MC-CDMA and/or MC-DS-CDMA systems have been published in [10, 11]. The BER (bit-error rate) analysis of an MC-CDMA based on considering different kinds of assumptions, to date, has been the focus of many previous researches [15, 26, 35]. The performance evaluation of MC-CDMA system over multipath fading channels was studied by Sourour and Nakagawa [26]. The results presented by Kim et al. [15] are for an uplink channel using MRC (maximal ratio combining) with an assumed frequency offset condition in correlated fading. The performance of MC-CDMA in non-independent Rayleigh fading was studied by Park et al. [22]. In a study by Shi and Latva-aho [29], which the CF (characteristic function) method and a residue were used calculation for downlink MC-CDMAs. Both the effect of the envelopes and phase correlation were considered by Shi and Latva-aho, who evaluated the performance of an MC-CDMA system operating in Rayleigh fading channel [30]. Aingwa Li and Latva-aho considered the error probability for in MC-CDMA systems [20] assumed that the transmission channel is in Nakagami-m fading, and the postdetection of EGC (equal gain combining) is considered. A BER analysis for an MC-CDMA system working in an imperfect channel estimate was conducted by Chong and Milstein [5]. The BER of an MC-CDMA system compared between Nakagami-m and Rayleigh fading channels was presented by Kang and Yao [17]. The same researchers [16] also analyzed the performance of MC-CDMA system over frequency-selective Nakagami-\(m\) fading channels with correlated and independent subcarriers. The authors, Xu and Milstein, in [31] proposed the performance evaluation of the MC-DS-CDMA system and considered working in the environments with the correlation in the fading of the various subcarriers and both MAI and PBI (partial-band interference) are considered. Yang and Hanzo [33] analyzed the performance of an MC-DS-CDMA considering the correlation presents in the fading of the various subcarriers. A unified approach was proposed by the same researchers [35] for evaluating the performance of an MC-DS-CDMA system over correlated-Nakagami-\(m\) channels. Recently, the present author Chen [2] evaluated the performance of an MC-DS-CDMA system with PBI working in Nakagami-\(m\) fading channels; whereas, Feng and Qin [8] obtained a simple approximate average BER expression for MC-CDMA systems over correlated-Nakagami-\(m\) fading channels. The error probability of an interleaved MC-CDMA system with the MRC receiver by adopting the methods of CF of the correlated Nakagami-\(m\) fading distributed was reported by Li and Latva-aho [36]. Furudate et al. [6] demonstrated that the system performance of an MC-CDMA system with a frequency interleaver is superior in various environments.

Nevertheless, the aforementioned publications almost were studied and considered under the scenarios with single-cell environment; however, in the reality, the stage of multiple-cell environments in wireless communication systems should be taken into account. Based on the motivation, a new designed construction configured by an MC-CDMA system operating within in both single- and multiple-cell environments is proposed in this paper. Moreover, compared to Rayleigh fading, we apply the Nakagami-\(m\) distribution, which can provide with a more and versatile way to wireless channels, to analyze the effect of the diversity branch correlation under the scenario that assumes the MC-CDMA system is working within single- and multiple-cell cases. On the other hand, this paper aims on the investigation of the effect of branch correlation which was considered as having an arbitrary correlation coefficient, for an MC-CDMA system with an MRC receiver operating over a correlated-Nakagami-\(m\) fading channel. According to the results from this study, it is...
necessary to stress that the impact of correlation phenomena between diversity branches is not only on the performance of single-cell MC-CDMA cellular systems, but does also on that of the situation with multiple-cell MC-CDMA cellular systems. Especially, the choice of an operating environment for the multiple-access wireless radio system does dominates the system performance of it.

This report is configured as follows. In Section II, an MC-CDMA system with MRC combination and a statistical model of a fading channel are presented; then, the statistical characteristics of the desired and the interference components are presented in Section III. The JPDF of the correlated fading envelopes is derived in Section IV. In Section V the average BER performance of the MC-CDMA system is evaluated, wherein a special case with a dual-branch is given as an example. Following the analytical results reported in Section V, a numerical manifestation and a brief discussion are presented in Section VI. Finally, in Section VII concludes the report.

II. SYSTEM MODELS OF MC-CDMA SYSTEM

1. MC-CDMA Transmitter Model

The generation of MC-CDMA signal is described in this section. If one assumes that there exist \( K \) simultaneously users with \( N \) subcarriers within a single cell, any effect of correlation among users will be ignored by assuming that the number of users is uniform within the distribution. As shown in Fig. 1, a single data symbol is replicated into \( N \) parallel copies. A signature sequence chip with a spreading code of length \( L \) is used for BPSK (binary phase-shift keying) to modulate each of the \( N \) subcarrier of the \( k \)-th user, where the subcarriers have frequency \( F/T_b \) Hz, and where \( F \) is an integer [4, 5]. The technical described previously is the same as to the performance of OFDM (orthogonal frequency division multiplexing) on a direct-sequence spread-spectrum signal when set \( F=1 \). The larger the value of \( F \), the more transmitting bandwidth is needed increase. The transmitted signal the results transmitted baseband signal \( S^{(k)}(t) \) corresponding to the \( M \) data bit size can be expressed as [34]

\[
S^{(k)}(t) = \sqrt{2P} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a^{(k)}[n]b^{(k)}[m]P_{T_k}(t-nT_b) \cos[2\pi(f_c+nF/T_b)t]
\]

where \( P \) is the power of data bit, \( M \) denotes the number of data bit, \( N \) denotes the number of subcarriers, and the sequencer \( a^{(k)}[0], \ldots, a^{(k)}[N-1] \) and \( b^{(k)}[0], \ldots, b^{(k)}[M-1] \) represent the signature sequences and the data bits of the \( k \)-th user, respectively, both \( a^{(k)}[n] \) and \( b^{(k)}[m'] \) belong to \([-1, 1]\). The term \( P_{T_k}(t) \) is defined as the unit amplitude pulse that is non-zero in the interval of \([0, T_b] \), and \( a_k=2\pi f_c+nF/T_b \) is the angular frequency of the \( n \)-th subcarrier, where \( f_c \) indicates the carrier frequency, \( T_b \) is the symbol duration.

2. MC-CDMA Receiver Model

A new designed block diagram of the receiver of an MC-CDMA system within the cells is shown in Fig. 1. When the code synchronization and/or acquisition are assumed to be accomplished completely for the first path of the desired signal, the received signal at the output of the referenced user can be determined by substituting (1) with the phase shift \( \phi_n^{(k)} = \phi_n^{(k)} - \phi_n^{(k)} \), of the channel of the \( k \)-th user and obtained as

\[
r(t) = \sum_{k=0}^{K-1} b_n^{(k)} S^{(k)}(t + \phi_n^{(k)}) + n(t)
\]

\[
= \sqrt{2P} \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} a_n^{(k)}(n) b_n^{(k)}[m] P_{T_k}(t-nT_b + \phi_n^{(k)}) \\
\times \cos[\omega_n t + 2\pi nF/T_b(t + \phi_n^{(k)})] + n(t)
\]

where \( \phi_n^{(k)} = \phi_n^{(k)} - \phi_n^{(k)} \) represents the phase difference between the transmitter and receiver, \( n(t) \) is the AWGN (additive white Gaussian noise) with a double-sided power spectral density of \( N_0/2 \). Assume that the acquisition of spreading code has been accomplished for the user of interest \((k=0)\), due to MRC diversity is considered, accordingly a perfect phase correction can be obtained, i.e., \( \hat{\theta}_{0,n} = \theta_{0,n} \).

With all the assumptions of MRC combining, the decision variable \( s^{(0)} \) of the \( n \)-th data bit for the referenced user is given by

\[
s^{(0)} = \sqrt{N_0 T_c} r(t) P^{(0)} n(t) \cos(\omega_n t + \phi_n^{(0)}) dt
\]

\[
= Z^{(0)} + I_{SI}^{(0)} + I_{ML}^{(0)} + I_{AWGN}^{(0)}
\]

where \( r(t) \) is the received signal shown in (2) for a single cell, and \( T_c \) denotes the chip duration. The first term of the previous equation represents the desired signal \( Z^{(0)} \) of the reference user, the second term, \( I_{SI}^{(0)} \), is the SI (self-interference) which results from the imperfect auto-correlation characteristic of spreading code, and will be assumed negligible in this study by carefully choosing the PN (pseudo-noise) code of the
MC-CDMA system; $I_{MAI}^{(0)}$ is the MAI (multiple-access interference) caused by the $(K-1)$ other simultaneous users in the system; the last term, $I_{AWGN}^{(0)}$, is the AWGN with zero mean and $N_0 T_b / 4$ variance.

3. Correlated-Channels Model

The received SNR (signal-to-noise ratio) on the $n$-th branch of the MRC receiver is expressed as $X_n = \beta_n^2 E_b / N_0$, where the weight, $\beta_n$, $n=0, 1, ..., N-1$, is modeled as Nakagami-$m$ distribution [21] for the $k$-th user, (the super fix $(k)$ is omitted), $E_b=P T_b$ denotes the energy per bit. It can be shown that the pdf (probability density function) of $X_n$ is Gamma-distributed with $\lambda_n = E_b \Omega_n / m_n N_0$, where $\Omega_n = E(\beta_n^2)$ is the average power of the fading signal, and $m_n = E(\beta_n^2) / E((\beta_n^2-\Omega_n)^2) \geq 0.5$ denotes the fading figure (parameter) of the amplitude distribution and characterizes the severity of channel fading. The equivalent fading figure will be assumed in the sequel, i.e., $m_k = m$ for $i \neq j$, where $i, j = 0, 1, ..., N-1$. Furthermore, if $\alpha_n = X_n / \lambda_n$ is set as the normalized SNR, then the pdf of $\alpha_n$ can be determined by simply changing the random variables and obtained as

$$P_{\alpha_n}(\alpha_n) = \frac{m_{n-1}}{\Gamma(m_n)} \exp(-\alpha_n)$$

The configuration of multiple-cell and cell geometry are adopted to the one which proposed by Fong et al. [7]. As shown in Fig. 2 for a multiple-cell system, where the referenced cell is numbered 0, the MAI induced from all the six adjacent cells (labeled with number 1 to 6 in Fig. 2) of the first

Fig. 1. Receiver block diagram of an cellular MC-CDMA system
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![Configuration of cell geometry](image)

Fig. 2. The structure of proposed cellular system

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![Configuration of cell geometry](image)

Fig. 2. The structure of proposed cellular system

tier and all the six further cells (labeled with number 7 to 12 in Fig. 2) will be taken into account in a multiple-cell MC-CDMA cellular system. For the sake of simplification in analysis, it is assumed that each BS (base station) is located at the center of its own cell and that the referenced mobile user is equally likely to be positioned anywhere within the reference cell. Accordingly, the received power at the reference mobile user will be affected by both lognormal shadowing and path loss, which is given as

\[ P_d = 10\log_{10} (d_g^{-\gamma}) \cdot P_k \]

where \( P_k \) expresses the power of one user in the referenced cell, \( \zeta_g \) denotes a Gaussian random variable with zero mean and standard deviation \( \sigma_g \), \( d_g \) indicates the distance between the \( g \)-th BS and the referenced mobile user, and \( \gamma \) is the path-loss exponent which depends on the media of the transmission [25].

Moreover, an exponential decaying MIP (multipath intensity profile) is assumed in this study. The MIP of the multiple-cell environment can be expressed as

\[ \Omega_{gn}^{(k)} = \Omega_{g0}^{(k)} \cdot e^{-\delta_g n}, \quad \Omega_{g0}^{(k)} \]

where \( \Omega_{g0}^{(k)} \) and \( \Omega_{g0}^{(k)} \) represent the \( n \)-th and the first path signal intensities in the \( g \)-th cell channel, respectively, and \( \delta_g \) is the rate of average power attenuation for the \( g \)-th cell signal propagation. In the following discussion it is assumed that \( \delta_g = \delta \)

for \( g = 0, 1, \ldots, G-1 \), where \( G \) indicates the total number of cells.

For considering the fading parameter, \( m \), of the Nakagami-\( m \) distribution, it is the same for every branch in the channel. The second moment of a correlated signal sequence within a single-cell can be expressed as [3]

\[ E[\sum_{n=0}^{N-1} \beta_n^{(k)}]^2 = 2N \sum_{n=0}^{N-1} (\Omega_n^{(k)})^2 \]

\[ + \left[ 2N(N-1) \cdot \frac{\Gamma^2(m+0.5)}{\Gamma^2(m)} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot m; \lambda_g \right] \]

where \( \beta_i \) represents the Gaussian hypergeometric function, and \( \lambda_g \) is the branch power correlation coefficient between \( \chi_i^{(k)} \) and \( \chi_j^{(k)} \), and is given as

\[ \lambda_g = \frac{\{E[x_i x_j] - E[x_i]E[x_j]\}}{\{(E[x_i]^2 - E^2[x_i])(E[x_j]^2 - E^2[x_j])\}^{1/2}}, \]

\[ i, j = n, 1, \ldots, N \]

where the superscript \( (k) \) is omitted. The normalized covariance matrix, \( C_M \), has the elements included in the set of \( \lambda_g \), \( i, j = 0, 1, \ldots, N \). However, it is more suitable to adopt the envelope correlation coefficient, \( \rho \), of \( \beta_i \) and \( \beta_j \), and which is given as

\[ \rho_g = E[\beta_i \beta_j] = \Phi(m, 1) \left[ 2F_{1}(-\frac{1}{2}, -\frac{1}{2}, m; \lambda_g) - 1 \right] \]

where \( \Phi(x, y) = \frac{\Gamma^2(x + \frac{1}{2})}{\Gamma(x + y)\Gamma(x)\Gamma^2(x + \frac{1}{2})} \).

**III. Calculation of the Statistical Results**

First in order to compute the SNR for the signal at the output of the decision maker, some of the expectations and variances should be calculated. The statistical calculation for the MC-CDMA system within single-cell and multiple-cell environments will be analyzed separately below, and where the subscripts SC and MC are in terms of the single- and multiple-cell, respectively, through the paper.
1. In Single-Cell Environments

The average value of the desired signal, the first term shown in (3), for the reference user in a single-cell environment can be obtained by the expectation operating and determined as

\[
(S)_{SC} = \mathbb{E}[Z^{(0)}] = \frac{P}{2N} \mathbb{E} \left[ \sum_{n=0}^{N-1} (\beta_n^{(0)})^2 \right] \tag{8}
\]

In this study, the self-interference shown in the second term of (3), \(I_{SI}^{(0)}\), is assumed to be ignorable by carefully choosing the PN sequence of the MC-CDMA system in practice. Then the variance of the total interference of the reference user in MC-CDMA system working in single-cell, \((\sigma^2)_{SC}\), of the referenced user in an MC-CDMA system working in a single-cell can be obtained as (the derivation is shown in Appendix I)

\[
(\sigma^2)_{SC} = \mathbb{E}[\text{Var}(I_{MAI}^{(0)})_{SC}] + \mathbb{E}[\text{Var}(I_{AWGN})_{SC}]
\]

\[
= \frac{P}{6N} (K-1) \mathbb{E} \left[ \rho_n^{(k)} \right]^2 \sum_{n=0}^{N-1} (\beta_n^{(0)})^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} \mathbb{E} \left[ \rho_n^{(0)} \right]^2 \tag{9}
\]

where \(\mathbb{E}()\) denotes the variance operator, \((I_{MAI}^{(0)})_{SC}\) and \((I_{AWGN})_{SC}\) are the MAI and AWGN term for an MC-CDMA system operating in a single-cell, respectively. The average power of the signal carried by the \(n\)-th subcarrier of the referenced user will be replaced with \(\mathbb{E}[(\rho_n^{(k)})^2] = \Omega_n^{(0)}\) in Section VI. Thus, the total variance of the interference can be obtained by simplifying the previous equation, to become as

\[
(\sigma^2)_{SC} = \frac{P}{3N} \sum_{n=0}^{N-1} (K-1) \Omega_n^{(0)} \mathbb{E} \left[ \rho_n^{(k)} \right]^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} \Omega_n^{(0)} \tag{10}
\]

where as illustrated in (5), the term \(\mathbb{E}[(\sum_{n=0}^{N-1} \beta_n^{(0)})^2]\) has included the correlation between the branches.

2. In Multiple-Cell Environments

The MC-CDMA system with a base station-to-mobile link operating in a multiple-cell cellular system is described in this subsection. It is reasonable to assume that there are \(K_g\) additional users locating in adjacent cell including in the first tier and the second tier, and they will induce an extra amount of MAI for the user locating in the local cell (labeled with \(\theta\) in Fig. 2). Without loss of generality, the total number of cells in the first and the second tier should be taken into account; i.e., the cell number is considered to be \(G=12\) (excluding the referenced one), which contains the six cells allocated in the first tier; however, in this study there are assumed to be six further cells in the second tier, as illustrated in Fig. 2. Hence, for the purpose of counting all the extra factors, the MAI term, \(\text{Var}(I_{MAI}^{(0)})_{SC}\), of the local cell is added by a second term which includes the combined MAI from cell 1 to cell \(G-1\) which is the cell with indexed 12 under the assumption. After summing all the contributions from the various cells, the variance, \((\sigma^2)_{MAI}\), of the MAI for the referenced user of the MC-CDMA system among the multiple-cell environment is computed by summing all the corresponding MAI terms come from the single-cell together and expressed as

\[
(\sigma^2)_{MAI} = \sum_{g=0}^{G-1} [(\sigma^2)_{MAI}]_{SC}
\]

\[
= \sum_{g=0}^{G-1} \left[ \frac{P^2}{12N^2} \sum_{k=1}^{K_g} \mathbb{E}[\rho_n^{(k)}]^2 \sum_{n=1}^{N-1} (\beta_n^{(0)})^2 \right]
\]

\[
= \frac{T_b^2 P^2 K_g^{-1}}{6N} \sum_{k=1}^{K_g} \sum_{n=1}^{N-1} \Omega_n^{(0)} + \sum_{g=1}^{G-1} A_g T_b^2 \frac{P K_g^{-1}}{6N} \sum_{k=1}^{K_g} \mathbb{E}[\rho_n^{(k)}]^2 \tag{11}
\]

where \(K_g\) and \(P\) denotes the number of user and the received attenuated signal power for the \(0\)-th cell, respectively; the average power of the referenced user is considered as \(\mathbb{E}[\rho_n^{(0)}] = \Omega_n^{(0)}\), in order to avoid confusion with the PSD (power spectral density) of the AWGN term, the subcarrier of the \(0\)-th cell is presented as \(N_{g=0}\) for which the average power assumed in each cell is equivalent, i.e., \(\mathbb{E}[\beta_n^{(0)}] = \Omega_n\), and \(A_g\) indicates the attenuation factor which is induced from the \(g\)-th \((g \neq 0)\) BS, which is given as [4]

\[
A_g = \left\{ \frac{d_g}{d_0} \right\}^{-\gamma} \frac{\left( \frac{d_g}{d_0} \right)^{\gamma}}{10}
\]

where cell \(g\) denotes any one of the six adjacent cells in the first tier and the six further cells in the second tier, as shown in Fig. 2, in the first tier, and \(R\) and \(\Theta\) represent the cell radius and the angle between the referenced mobile and the referenced BS, respectively. Obviously, the first term in (11) denotes the MAI contributed by all other user within the referenced cell and the second term is the MAI contributed by all users located on
other adjacent cells in the first tier, respectively. Note that the term \( E[\gamma^2] \) in (11) is obtained by assuming that the branches within the referenced cell are statistically dependent; whereas, those from the interfering \( G \) cells are reasonably assumed to be independent as long as the separation between cells is large enough.

After summing all the variances of AWGN generated from the surrounding single-cell together and assuming the macro diversity doesn’t involved, then the variance of the AWGN for the multiple-cell, \( \sigma^2_{\text{AWGN}}^{MC} \), can be calculated as

\[
(\sigma^2_{\text{AWGN}})^{MC} = \sum_{g=0}^{G-1} (\sigma^2_{\text{AWGN}})
\]

\[
= \frac{G-1}{4T_b} \sum_{g=0}^{G-1} \frac{N_0}{4T_b} E[(\beta_g^0)^2] = \frac{N_0 \sigma^2}{4T_b} \quad (13)
\]

Hereafter, by summing (10), (11) and (13), the total variance \( \text{(var)}^{MC} \) for the referenced user of the MC-CDMA system within the multiple-cell can be determined as [31]

\[
\text{(var)}^{MC} = [\text{var}]_{\text{MC}} + \text{var}_{\text{MC}} + \sigma^2_{\text{AWGN}}^{MC}
\]

\[
= \frac{\Omega_0^2 \beta_0^2}{2} \sum_{l=0}^{L-1} (\gamma_l^{(1)})^2 \left( \frac{K - 1}{3N} \times \sum_{l=1}^{L} \sum_{l=1}^{L} e^{-i\delta/L} \right) + \left( \frac{2L(L-1)\Gamma^2(m+0.5)}{\Gamma(m)} \sum_{l=0}^{L-1} \frac{\Omega_l^2}{m} \right)^{1/2} \left( \frac{2L(L-1)}{\Gamma^2(m+0.5)} \sum_{l=0}^{L-1} \frac{\Omega_l^2}{m} \right)^{1/2} e^{-\delta(L-1)/2} + \frac{K}{N} \times 2 \times 0.02409 \times \sum_{l=0}^{L-1} \frac{e^{-i\delta}}{2E_b\Omega_0} + \frac{N_0}{2E_b\Omega_0}
\]

(14)

where except for the reference cell, \( g=0 \), the cell number is assumed to be \( G=12 \), i.e., \( g=1, 2, \ldots, 12 \), including the cells allocated in the first tire; however, there are six other cells further in the second tier, as shown in Fig. 2.

**IV. CALCULATION OF THE JOINT PDF**

In this section, the JPDF of the total power on the MRC output of the referenced user is determined. The SNR at the output of the MRC receiver, which is a sum of the squares of the signal strength of the \( i \)-th branch \( \beta_i \) is given by

\[
\gamma = \sum_{l=0}^{L-1} \beta_i^2
\]

(15)

where \( \beta_i \) follows the Nakagami-\( m \) distribution. Firstly, since the pdf of \( \gamma \) must be calculated to determine the average BER of the MC-CDMA system, one may now determine the joint characteristic function of \( \gamma \), which can be obtained as [28]

\[
\phi_j(j_{t0}, \ldots, j_{t(L-1)}) = E[\exp\{j(t_0\beta_0^2 + \ldots + t_{(L-1)}\beta_{(L-1)}^2)\}]
\]

\[
= 1/ \det(I - jTU)^m
\]

(16)

where \( T=\text{diag}\{\tau_0, \tau_1, \ldots, \tau_{L-1}\} \) is the diagonal matrix with element \( \tau_0, \tau_1, \ldots, \tau_{L-1} \), and \( U \) is the positive definite matrix specified by the power branch covariance matrix \( C_\gamma \) with the elements of branch power correlation coefficients as shown in (6).

In order to determine the JPDF of the correlated fading channels, the Equation (16) is expanded and \( j_t \) is replaced with \( s_0, l=0, 1, \ldots, L-1 \); then the joint characteristic function is expressed as

\[
\phi_j(s_0, \ldots, s_{L-1}) = (1-s_0)(1-s_1) \cdots (1-s_{L-1}) \cdot G(a_0, a_1, \ldots, a_{L-1})
\]

(17)

where \( s_i=\Omega_\gamma \), \( a_l = -s_l/m(1-s_l) \), and

\[
G(a_0, a_1, \ldots, a_{L-1}) = 1 - \left( \sum_{l<j} V_{l,j} \cdot a_l a_k + \sum_{l<j<k} V_{l,j,k} \cdot a_l a_j a_k + \ldots \right) + \text{det} V_{0,1, \ldots, (L-1)} \cdot a_0 a_1 \cdots a_{(L-1)}
\]

(18)

where the term \( V_{0,1, \ldots, (L-1)} \) is the determinant of the covariance matrix \( C_\gamma \), which is usually written as

\[
V_{0,1, \ldots, (L-1)} = (-1)^L \cdot \text{det} \begin{bmatrix} 0 & \rho_{01} & \cdots & \rho_{0(L-1)} \\ \rho_{01} & 0 & \cdots & \rho_{0(L-1)} \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{(L-1)0} & \rho_{(L-1)1} & \cdots & 0 \end{bmatrix}_{L \times L}
\]

(19)

The term \( V_{0,1, \ldots, (L-1)} \) is used to measure the correlation among the diversity branches, and \( \rho_i \) has the same definition as in (7). By applying the Maclaurin’s theorem, (17) can be
further expanded as

\[ \phi_x(s_0, \ldots, s_{N-1}) \]

\[ = \sum_{n=0}^{(m/2)_u \text{ mod } m} n! \left(1 - \varepsilon \right)^{-n} \cdot \sum_{y=0}^{L-1} \left( \sum_{x=0}^{L-1} (a_x)^y \right)^n \]

(20)

where \((m)_u = (m+1) - (m+u-1) - \Gamma(m+u)/\Gamma(m)\) denotes the Pochhammer symbol, \(y\) is a polynomial in \(\psi_{L-1}, \ldots, \psi_0\) and \(\varepsilon\), \(v = 0, \ldots, (L-1)\) are nonnegative integers of which not more than \((L-2)\) are zeros.

Then the inverse Laplace transform is taken for (20) by means of the properties of the generalized Laguerre polynomial given as [26, (1.3)]

\[ f_{\psi L} e^{x^2} H_{\psi}(s, y) ds = \left( \frac{x}{1-x} \right)^y \left( \frac{1}{(1-x)^y} \right) \]

(21)

where \(H(s, y) = \psi L(s) L_{\psi}(s, y) / y\), \(\psi L(s)\) is a marginal pdf of random variable \(s\), and \(L_{\psi}(s, y)\) is the generalized Laguerre polynomial of degree \(r\), defined as

\[ L_{\psi}(s, y) = (s+1)^{r+1} / \Gamma(r+1) \]

\[ \cdot \left( \frac{d}{ds} \right)^{r+1} \left( \frac{s^{2r+1}}{(1-s)^{r+1}} \right) \]

(23)

Now, it is known that the definition, \(\lambda = \varepsilon L_{\psi} / m\), \(\alpha_t\), and \(X_i = \beta_i^2 E_i / N_0\), the normalized SNR for each diversity branch, can be expressed as

\[ \alpha_t = m \beta^2 / \Omega_{\delta}^2, \ldots, \alpha_{(L-1)} = m \beta^2_{(L-1)} / \Omega_{\delta(L-1)}^2 \].

Hence the generalized PDF of the normalized SNR, \(f_{\alpha_t, \ldots, \alpha_{(L-1)}}(m \beta^2 / \Omega_{\delta}^2, \ldots, m \beta^2_{(L-1)} / \Omega_{\delta(L-1)}^2)\), can be obtained as

\[ f_{\alpha_t, \ldots, \alpha_{(L-1)}}(\alpha_t, \ldots, \alpha_{(L-1)}) \]

\[ = \prod_{l=0}^{L-1} P_{\phi}(\alpha_t) \cdot \sum_{k=0}^{(m/2)_u} \frac{(m!)^2}{k!} \sum_{i=0}^{L-1} \left[ \frac{L(\alpha_t, m)}{m} \cdot \frac{L(\alpha_j, m)}{m} \right] + \ldots \]

\[ + V_{0, \ldots, (L-1)} \left[ \frac{L(\alpha_0, m)}{m} \ldots \frac{L(\alpha_{(L-1)}, m)}{m} \right]^h \]

(24)

where the \(h\)-th power of the Laguerre polynomial divided by \(m\) can be written as [36]

\[ \left[ \frac{L(x, m)^h}{m} \right] = \frac{L_h(x, m)}{(m)_h} \]

(25)

Finally, by replacing the term \(P_{\phi}(\alpha_t)\) in (24) with (4), the JPDF of \(L_{\psi}\) correlated with Nakagami-\(m\) random variables can be obtained as

\[ f_{\alpha_t, \ldots, \alpha_{(L-1)}}(\alpha_t, \ldots, \alpha_{(L-1)}) \]

\[ = \prod_{l=0}^{L-1} (\alpha_t, \ldots, \alpha_{(L-1)}) \cdot \sum_{k=0}^{(m/2)_u} \frac{(m!)^2}{k!} \sum_{i=0}^{L-1} \left[ \frac{L(\alpha_t, m)}{m} \cdot \frac{L(\alpha_j, m)}{m} \right] + \ldots \]

\[ + V_{0, \ldots, (L-1)} \left[ \frac{L(\alpha_0, m)}{m} \ldots \frac{L(\alpha_{(L-1)}, m)}{m} \right]^h \]

(26)

where \(\{\ldots\}^h\) is a symbol of the \(h\)-th power of a multinomial.

V. EXAMPLE AND BIT ERROR PROBABILITY

Consequently, with coherent demodulation, the BER conditioned on the instantaneous signal amplitude \(\alpha_t\) for an MC-CDMA system working in single-cell or multiple-cell is given as [27]

\[ P_{\text{MC-CDMA}}(\alpha_t, l = 0, 1, \ldots, L-1) = Q(\sqrt{\text{SNR}}) \]

\[ = Q \left( \frac{\left( \frac{\lambda^2}{2} \right)_{\text{SC or MC}}}{\left( \frac{\sigma^2}{2} \right)_{\text{SC or MC}}} \right) \]

(27)

where \(\left( \frac{\lambda^2}{2} \right)_{\text{SC or MC}} \) and \(\left( \frac{\sigma^2}{2} \right)_{\text{SC or MC}} \) represent the mean value of the desired signal and the total interference of the single-cell or multiple-cell, respectively, and \(Q(t)\) is the well known Marcum’s \(Q\)-function, which can be alternative expressed as [19]

\[ Q(t) = \frac{1}{\pi} \int_{0}^{\pi} \frac{e^{-\frac{t^2}{2}}}{2 \sin^2 \theta} d\theta \]

(28)

Once the desired signal and the total interference are determined, the average error probability for an MC-CDMA system in correlated-Nakagami-\(m\) fading channels can be
accomplished by averaging $P_e(error|\alpha, l=0, 1, ..., L-1)$ over $L$ variates with the JPDF shown in (27), and it can be calculated as

$$P_{av} = \prod_{l=0}^{L-1} P_e(error|\alpha_l, l=0, 1, ..., L-1) \times f_{\alpha_0, ..., \alpha_{L-1}}(\alpha_0, ..., \alpha_{L-1}) \cdot d\alpha_0 \cdot d\alpha_1 \cdot \cdots \cdot d\alpha_{L-1}$$

(29)

The previous equation involving $L$-fold integration can be evaluated by the detail means given in Reference [27].

1. Dual Correlated Branch Example

To validate the accuracy of the derived formula, an example with dual-branching is presented in this subsection. The JPDF with a correlated Nakagami-$m$ fading channel for a dual-branch can be obtained by putting $\alpha_0$ and $\alpha_1$ into (24), become as

$$f_{\alpha_0, \alpha_1}(\alpha_0, \alpha_1) = P(\alpha_0) \cdot P(\alpha_1) \cdot \sum_{h=0}^{\infty} \frac{(m/2)_h}{h!} (V_{01})^h \cdot \left(\frac{L(\alpha_0, m)}{m} \cdot \frac{L(\alpha_1, m)}{m}\right)^h \cdot d\alpha_0 \cdot d\alpha_1$$

(30)

where $P(\alpha_i)$, $i=0, 1$ are shown in (4), for which the probability of bit error using BPSK modulation conditioned on the instantaneous signal amplitude $\beta$ for the MC-CDMA system working in multiple-cell can be expressed as [27]

$$P_e(error|\beta, \gamma_{0,1}, l=0, 1, ..., L-1) = Q(\sqrt{SNR}) = Q\left(\frac{\mu_{MC}^2}{\sigma^2_{MC}}\right)$$

(31)

and subsequently integrated over the conditional BER pdf shown in (30). The average BER, $P_{av}$, for the MC-CDMA system with dual-branching case becomes

$$P_{av} = \int_0^\infty \int_0^\infty Q(\sqrt{SNR}) \cdot f_{\alpha_0, \alpha_1}(\alpha_0, \alpha_1) \cdot d\alpha_0 \cdot d\alpha_1$$

$$= \int_0^\infty \int_0^\infty \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(\alpha_0^2 + \alpha_1^2)} \cdot d\alpha_0 \cdot d\alpha_1$$

$$= \frac{m}{\sqrt{\pi}} \cdot \sum_{h=0}^{\infty} \frac{(m/2)_h}{h!} (V_{01})^h \cdot \left(\frac{L(\alpha_0, m)}{m}\right)^h \cdot d\alpha_0$$

(32)

where the mathematical equivalent of the Marcum’s Q-function can be alternative replaced with the expression shown in (28). By interchanging the order of integration shown in (32), so that the equation can be determined as

$$P_{av} = \frac{1}{\pi} \sum_{h=0}^{\infty} \frac{(m/2)_h}{h!} (V_{01})^h \int_0^\infty e^{-\frac{1}{2\sigma^2}(\alpha_0^2 + \alpha_1^2)} \cdot d\alpha_0$$

$$= \int_0^\infty \exp\left(-\frac{1}{2\sigma^2}(\alpha_0^2 + \alpha_1^2)\right) \cdot \left(\frac{L(\alpha_0, m)}{m}\right)^h \cdot d\alpha_0$$

(33)

and the subscript of $\alpha_0, \alpha_1 = 0, 1$, is deleted for the purpose of simplification, and $P(\alpha)$ represents the Gamma pdf shown in (4). By putting the integral terms into the brackets of (33), the integral terms can be calculated as

$$= \int_0^\infty \exp\left[-\frac{1}{2\sigma^2}(\alpha_0^2 + \alpha_1^2)\right] \cdot \left(\frac{L(\alpha_0, m)}{m}\right)^h \cdot \frac{\Gamma(m)}{\Gamma(m)\Gamma(m)} \cdot \left(\frac{\Gamma(m)}{\Gamma(m)}\right)$$

(34)

where some of the mathematically equivalent formulas listed in [1] have been employed.

Thus, the average BER of MC-CDMA system with dual correlated branch operating in a multiple-cell environment has
been determined as shown in Equation (33). Based on the result (33), the phenomenon of correlation between the different branches will be illustrated by means of a numerical analysis. A quasi-Gaussian correlation model of an equally spaced linear array with an arbitrary correlation coefficient is adopted, being given as [18]

\[ \lambda_{ij} = \exp[-0.5\eta(i - j)^2 \left( \frac{d}{\lambda} \right)^2], i, j = 0, ..., L - 1 \]  

(35)

where \( \eta \approx 21.4 \) is a coefficient chosen from setting this correlation model equal to the Bessel correlation model with a -3 dB point [19], and \( d/\lambda \) is the normalized distance between two neighboring branches, with \( \lambda \) denoting the wavelength of the carrier frequency. The parameter \( d/\lambda \) is applied to determine the threshold level of correlation. The assigned values of \( d/\lambda \) are 0, 0.1, 0.2, and \( \infty \), in which \( d/\lambda = 0 \) and \( d/\lambda \to \infty \) represent two extreme conditions, i.e., fully correlated and uncorrelated branches, respectively.

VI. RESULTS AND DISCUSSIONS

In this section, the previous analytical results are examined by an intensive numerical computer evaluation and subsequently discussed. Since the reasons of considering a downlink mobile radio communication channel and the auto-correlation characteristic of the spreading codes \( S (0)_i \) is neglected in (3) based on the PN sequence is chosen appropriately for the MC-CDMA system, the Hadamard Walsh codes are utilized as an optimum orthogonal set in this analysis. Besides, the conditions of perfect CSI (channel side information) and code with complete synchronization are also considered. The BER compared in an MC-CDMA system operating with different subcarriers, \( N=256 \) and \( N=512 \), for varying \( d/\lambda \) values at Nakagami-\( m \) parameter \( m=2 \), user numbers \( K=20 \), diversity branches \( L=2 \), and the rate of average power attenuation \( \delta=0.05 \), is shown in Fig. 3. It is obviously to discover that when an MC-CDMA system is with fewer subcarrier will obtain much more inferior system performance. However, merely increasing the number of subcarrier cannot promote a distinctive performance, the reason being that the amount of interference is caused by the other user within both the same cell or other cells, i.e., the extra interference, \( \text{VAR}_{MCMAI} \), is included in the total interference. Accordingly, the results of this phenomenon are due to the impact of the branch correlation characteristic. In Fig. 4, dual-branch correlation is assumed, illustrates the BER curves of the MC-CDMA system in a multiple-cell case with the parameters of, \( K=20, N=512 \), and \( \delta=0.05 \) at different \( d/\lambda \) values, which are varied with 0.15, 0.3, and 0.9, for the Nakagami-\( m \) fading parameters \( m=2 \), and \( m=5 \), respectively. As the value of \( d/\lambda \) decreases, the performance of the MC-CDMA system is insignificantly deteriorated. The fact of the limitation of interference can also be observed in that further SNR increase is useless for system BER performance. It is also worth to note that the system performance becomes more superior with a larger value of the Nakagami-\( m \) fading parameter. A comparison of user capacity for an MC-CDMA system operating in multiple-cell, with subcarrier number \( N=512 \), and \( \delta=0.05 \), the bit SNR is fixed and assumed to be 10dB, is illustrated in Fig. 5. The results from a comparison of the Nakagami-\( m \) fading parameter and the \( d/\lambda \) values are also shown in the same figure. All the illustrated curves are reasonable, compared with the expected facts. However, it is valuable to observe that the user capacity is affected more by the Nakagami-\( m \) fading parameter than that of the \( d/\lambda \) value when the MC-CDMA system is working in
multiple-cell situation. Especially, the fact is worthwhile noting that the system performance of MC-CDMA system in multiple-cell situation will become inferior after the user number greater than about 10 users if the conditions of a cellular system without appropriately well designed. Definitely, it can be noted that the decision of environmental specification, which associates with the Nakagami-\( m \) fading figures (\( m \) values), of the BS required for an MC-CDMA system operating in a cellular environment are the most one important factor.

**VII. CONCLUSIONS**

The performance of an MC-CDMA system in a single-cell or multiple-cell environment with a correlated-Nakagami-\( m \) fading channel has been theoretically evaluated with the JPDF, in which the generalized Laguerre polynomial was employed, and validated by a dual correlated-branch example. The effects of branch-power correlation and certain system parameters have been taken into account. The results show that the performance of an MC-CDMA system is sensitive to even small values of correlation coefficient of the correlated branch. In different operating environments, a single-cell or multiple-cell scenario, including the cells in the first tier and 6 further cells in second tier, has been considered in this paper. Since interference will come from the cells, the overall performance of the MC-CDMA system in the former case is superior to that of the latter. In addition, the results show that the fact of an increase in an MC-CDMA system due to the bit SNR will approach a limitation since the system’s performance will be limited by interference, whether from the same BS of other users located at the other cells.

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**REFERENCES**

In this appendix we derive the total interference, \((\sigma_F^2)_{SC}\), of the 0-th user of the MC-CDMA system in single-cell. By utilizing the variances formula and then summing up the AWGN term and the MAI term, such that

\[
(\sigma_F^2)_{SC} = \left[\text{Var}(I_{\text{AWGN}}^{(0)}_{SC}) + \text{Var}(I_{\text{MAI}}^{(0)}_{SC})\right]
\]

\[
= \left[\frac{P}{2N} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_n^{(0)} b_m^{(k)} \alpha_n^{(0)} \beta_m^{(k)} \cos(\omega_n t + \phi_n^{(0)})\right] + \text{Var}\left[\sum_{n=0}^{N-1} \frac{N_0}{4T_b} (\beta_n^{(0)})^2\right]
\]

\[
= E\left[\left(\frac{P}{2N} \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_n^{(0)} b_m^{(k)} \alpha_n^{(0)} \beta_m^{(k)} \cos(\omega_n t + \phi_n^{(0)})\right)^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} (\beta_n^{(0)})^2\right]
\]

\[
= \left(\frac{P}{2N}\right) \frac{T_b^2}{3N^3} \sum_{k=1}^{K-1} r_{k1}(N) \cdot E\left[\beta_n^{(k)}\right]^2 \left[\cos^2(\omega_n t + \phi_n^{(k)})\right] \sum_{n=0}^{N-1} (\beta_n^{(0)})^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} (\beta_n^{(0)})^2
\]

\]

(I.1)

where the correlation results, \(r_{k1}(N)=2N^2\), is employed [35], then the variance becomes as

\[
(\sigma_F^2)_{SC} = \frac{P}{2N} \frac{T_b^2}{3N^3} 2N^2 \frac{1}{2} \sum_{k=1}^{K-1} E\left[\beta_n^{(k)}\right]^2 \left[\beta_n^{(0)}\right]^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} (\beta_n^{(0)})^2
\]

\[
= \frac{P}{6N} (K-1) E\left[\beta_n^{(k)}\right]^2 \sum_{k=1}^{K-1} (\beta_n^{(0)})^2 + \frac{N_0}{4T_b} \sum_{n=0}^{N-1} (\beta_n^{(0)})^2
\]

(I.2)