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Applying Correlated-Gamma Statistics in the Evaluation of the Channel Capacity of a Dual-Branch MRC Diversity System

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ABSTRACT

The channel capacity of dual-branch MRC (maximal ratio combining) diversity system over correlated waveform intensity, which is characterized as correlated-Nakagami-*m* fading (the power is naturally modeled as the correlated-Gamma statistics), is evaluated in this paper. The formulas of channel capacity performance are provided with a pdf (probability density function)-based approach. The pdf of sum of Gamma variates based on the representation of a single gamma series is adopted in the paper. The corresponding expressions of the pdf with the Rayleigh distribution are able to be obtained as a special case of Nakagami-*m* fading. Finally, the numerical examples are presented and discussed for illustrating the purpose of validating the accuracy of the channel capacity equations derived in this paper.

***Key Words*:** channel capacity, correlated-Gamma variates, MRC diversity, Nakagami-*m* fading

應用相關伽碼統計於雙分支 MRC 分集之通道容量評估

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摘　要

於本論文中主要評估於本論文中主要評估雙分支最大比例合成（maximal ratio combining, MCR）分集系統之通道容量；其中衰落通道以相關中上衰落（correlated-Nakagami-*m* fading）統計為模式，自然地，其衰落功率利用模式化而成相關伽碼（correlated-Gamma）分布。本論文提供機率密度函數（probability density function, pdf）為基礎的通道容量效能之公式；其中在本文中採用單一 Gamma 級數為表示式的分佈所得之 pdf 結果可以經由 Nakagami-*m* 的特例而得。最後，為了認證所推導而得之通道容量公式的正確性，吾亦於本文中顯示及討論數值分析的例子。

關鍵詞：通道容量，相關伽碼變異，最大比例合成分集，中上衰落

I. INTRODUCTION

It is recognized that the mobile radio links are subject to severe multipath fading in the received signal envelope due to the combination of randomly delayed, reflected, scattered, and diffracted signal components. Such a fading will result in degradation in the capacity of diversity schemes. The channel capacity is one of the most important criterions for evaluating the system performance of the wireless radio systems. The channel capacity analysis of fading channel is gradually growing to as a primary benchmark adopted in either wired or wireless communication systems. Especially, the important issues of evaluating channel capacity are focused on calculating the SNR (signal-to-noise ratio) and the communication bandwidth. Since Shannon proposed the channel capacity theorem in 1948 [16], there are several papers have been published dealing with the channel capacity characteristic of the diversity combining techniques under fading environments (e.g., [2, 3, 6-8, 12, 17, 20]). Particularly, the capacity of Nakagami-*m* fading channels was investigated in [2, 4, 20, 21]. In [20], an expression for the channel capacity of Nakagami-*m* fading with MRC (maximal ratio combining) diversity was obtained with the assumption that the diversity branches are *i.i.d.* (independent and identically distributed). There is the block diagram of a dual-branch MRC diversity system is shown in Fig. 1. The expressions for the channel capacity of Nakagami-*m* fading channels with MRC were obtained in [7] under different adaptive transmission techniques for the *i.i.d.* case. The effect of branch correlation on the capacity of Nakagami-*m* fading channels for different diversity combining techniques was investigated. The diversity combining techniques are known to be a powerful technique that can be used to improve system performance under channel fading caused by the maneuver phenomena [18]. It is worthwhile to note that the pdf (probability density function) of the received SNR (signal-to-noise ratio) applied in this paper for deriving an expression of the channel capacity for MRC has been determined in [1] based on the work of Gurland [10], which is different but is equivalent to the pdf used in [21]. The corresponding expression for Rayleigh fading is able to be obtained as a special case of Nakagami-*m* fading. Assume that the channels are slowly varying flat correlated-fading channels. In the past, several models have been proposed to explain the behavior of multipath fading of the received signal envelope. It has been a long time that the Rayleigh and Rice distributions are employed to characterize the envelope of fading signals over small geographical area and/or short term fading. Recently, Nakagami-*m* distribution has been thoroughly investigated due to the fact that it is verified to be a more versatile model for a variety of fading environments such as the urban/suburban radio multipath channel for wireless communication systems, even for indoor propagation, and it includes the Rayleigh fading as a special case where the fading factor m=1 [14]. The other property of a multipath-fading channel is in its MIP (multipath intensity profile), which depicts the average power at the output of the channel as a function of path delay [11, 15]. The multipath propagation can be resolved at the MRC receiver by means of the wideband characteristics of the signal [19].

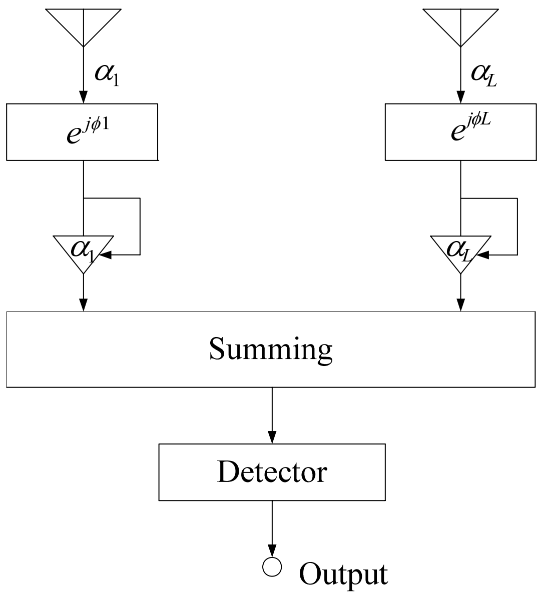

Fig. 1. The block diagram of a dual-branch MRC diversity system

After reviewing the publication mentioned above, there is a much simpler closed-form formula, in which both the fading parameter and the MIP factor are considered, for the channel capacity of a dual-branch MRC diversity is derived via a single variable Gamma pdf in this paper. On the other hand, the evaluation of channel capacity for a dual-branch MRC schemes over correlated Nakagami-*m* fading channels with integer fading parameter, m, and the MIP factor, u, are conducted. The power of the fading signal is considered to model as Gamma distributed with the characteristic of branch correlation. In order to verify the correction of the derived formulas, all results from the numerical evaluation are compared with the previous works described in the last paragraph. After the introduction section, the organization of this paper is constructed as follow, the receiver and channel models of the correlated-Gamma for the received signal at the output of the dual-branch MRC diversity are described in section II. In section III the evaluation of system performance of channel capacity is performed, and the results of the equation are numerical evaluated by the software package in section IV. Finally, in section V where a simple conclusion is presented

II. RECEIVER AND CHANNEL MODELS

In this section, the receiver of an dual-branch MRC diversity and the correlated channels models are described. Considering that the waveform intensities at the output of the dual-branch MRC scheme are r_i(t), i=1, 2, and the signal appears at the output of the dual-branch MRC diversity is the summation of the two branch waveforms, given as $r(t)=\sum_{i=1}^{2} r_i(t)$.

First, it is well known that the mobile radio channel is usually modeled as a discrete, slow-fading, and time-invariant multipath channel. The equivalent complex low-pass impulse response of the fading channel viewed from the i-th user is given by [5]

$$h_{(i)}(\tau)=\sum_{l=1}^{L}\beta_{(i)}^{l} e^{j\theta_{(i)}^{l}}\delta[\tau-\tau_{(i)}^{l}] \tag{1}$$

where β^l, θ^l and τ^l are the path signal intensity, phase, and delay of the l-th multipath, respectively. Now recall that the fact of definition of a SNR variable is going to be a gamma variate and adopt the representation of two key results for the summation of correlated gamma variates. Assume that the random variable X follows a gamma distribution with parameter $m > 0$ and $\beta > 0$, the pdf of X is then given as

$$P_X(x)=\frac{x^{m-1}e^{-x/\beta}}{\beta^m\Gamma(m)}U(x) \tag{2}$$

where $\Gamma(\cdot)$ is the gamma function [11], and $U(\cdot)$ denotes the unit step function. On the other hand, the shorthand notation of X can be represented as $X \sim \omega(m, \beta)$. In order to determine the pdf of the sum of correlated Gamma variates, now extend the Moschopoulos result and an exact single gamma-series representation of the sum of arbitrarily correlated gamma variates can be obtained [4]. Next, express X_n, n=1, 2,…, L as a set of L correlated-gamma variates with parameters m and β_n, respectively, [i.e., $X_n \sim \omega(m, \beta)$] and let ρ_{ij}, i=j=1, 2,…, L denotes the correlation coefficient between X_i and X_j, i.e.,

$$\rho_{ij}=\rho_{ji}=\frac{Cov(X_i,X_j)}{\sqrt{\mathrm{Var}(X_i)\mathrm{Var}(X_j)}},\ 0\le\rho_{ij}\le 1.\ i,j=1,2,\ldots,L \tag{3}$$

then the pdf of $Y=\sum_{n=1}^{L}X_n$ can be expressed as

$$P_Y(y)=\prod_{n=1}^{L}\left(\frac{\lambda_1}{\lambda_n}\right)^m\sum_{k=0}^{\infty}\frac{\delta_k y^{Lm+k-1}e^{-y/\lambda_1}}{\lambda_1^{Lm+k}\Gamma(Lm+k)}U(y) \tag{4}$$

where λ_1=min$_n\{\lambda_n$, n=1, 2,…,$L\}$, where min$_n\{\cdot\}$ is a function of choosing the minimum one, $\{\lambda_n$, n=1, 2,…, $L\}$ are the eigenvalues of the matrix $A=DC$, where D is the $L\times L$ diagonal matrix with the entries $\{\beta_n$, n=1, 2,…, $L\}$ and C is the $L\times L$ positive definite matrix defined by

$$C=\begin{bmatrix}1 & \sqrt{\rho_{12}} & \cdots & \sqrt{\rho_{1L}}\\ \sqrt{\rho_{21}} & 1 & \cdots & \sqrt{\rho_{2L}}\\ . & . & . & .\\ . & . & . & .\\ \sqrt{\rho_{L1}} & \cdots & \cdots & .\end{bmatrix}_{L\times L} \tag{5}$$

and the coefficients δ_k can be obtained recursively by the formula

$$\begin{cases}\delta_0=1\\ \delta_{k+1}=\dfrac{m}{k+1}\displaystyle\sum_{i}^{k+1}\left[\sum_{j=1}^{L}\left(1-\frac{\lambda_1}{\lambda_j}\right)^i\right]\delta_{k+1-i},\ k=0,1,2,\ldots\ldots\end{cases} \tag{6}$$

For constant correlation, it can be show that the eigenvalues of the matrix A can be defined as [15, (2.8.3)]

$$\begin{cases}\lambda_1=\ldots=\lambda_{L-1}=\dfrac{\bar{\gamma}}{m}\left(1-\sqrt{\rho}\right)\\ \lambda_L=\dfrac{\bar{\gamma}}{m}\left(1+\sqrt{\rho}(L-1)\right)\end{cases} \tag{7}$$

Once by substituting these eigenvalues in previous equation into (4), the statistical distribution of SNR at the combiner output can be obtained immediately. Thus, it is able to easily determine the channel capacity of the dual-branch MRC diversity over correlated-fading channels. Moreover, in order to take the MIP (multipath intensity profile) effect into account for the evaluation of channel capacity of the MRC diversity. Consider the condition of negative exponential MIP with power decay factor, μ, then the average power of the SNR can be computed as $\Omega_{j,n}=e^{-n\mu}/q(L,\ \mu)$, j=1, 2,…, M_R, n = 0, 1,…, L_R-1, where assuming that L is the number of multipath of the desired signal, and the function of $q(L, \mu)$ is defined as

$$q(L,\mu)=\sum_{l=0}^{L\text{-}1}e^{-l\mu}=\frac{1-e^{-L\mu}}{1-e^{-\mu}} \tag{8}$$

After the definition is completed, the eigenvalues of λ_L shown in (7) is going to be replaced with the equivalent shown as

$$\lambda_L=\frac{1}{m}\left(1+\sqrt{\rho}\left(L-1\right)\right)\bar{\gamma}\frac{e^{-n\mu}}{q(L,\mu)} \tag{9}$$

where the parameters of correlation coefficient, ρ, and MIP, μ, are all included in the scenario considered in this paper.

III. CHANNEL CAPACITY ANALYSIS

The channel capacity, which is also known as the ergodic capacity, of a radio system operating in flat fading channel can be obtained by averaging the capacity over an AWGN (additive white Gaussian noise) channel. The capacity which is defined as $C= B\log_2(1+\gamma)$, where γ represents the instantaneous SNR of the received signal at the output of a receiver with MRC scheme, and B denotes the channel bandwidth (Hz) [16]. Thus, the average channel capacity, $\langle C\rangle$, of a radio system in flat fading channel is expressed as [12]

$$\langle C\rangle=B\cdot\int_0^\infty \log_2(1+\gamma)p_\gamma(\gamma)d\gamma \tag{10}$$

where for the purpose of the random process reason, the determination of pdf, $P_\gamma(\gamma)$, for the instantaneous SNR is necessary. Moreover, because the evaluation of the channel capacity, η_{MRC}, is for a dual-branch MRC system over correlated Nakagami-*m* fading channels, the results can be calculated by substituting (4) into (10) and obtained as

$$\eta_{MRC}=\frac{\langle C\rangle}{B}=\prod_{n=0}^{L}\left(\frac{\lambda_1}{\lambda_n}\right)^m\sum_{k=0}^{\infty}\frac{\delta_k}{\lambda_1^{Lm+k-1}\Gamma(Nm+k)}$$
$$\times\int_0^\infty \log_2(1+\gamma)\gamma^{Lm+k-1}e^{-\gamma/\lambda_1}d\gamma \tag{11}$$

where an integral formula with the logarithm function is meeting in the previous equation. The partial integration method may be applied to obtain the final closed-form of the formula, and after some algebra procedures, the result cab be expressed as (see the Appendix A)

$$\eta_{MRC}=\frac{\langle C\rangle}{B}=\prod_{n=0}^{L}\left(\frac{\lambda_1}{\lambda_n}\right)^m\sum_{k=0}^{\infty}\frac{\delta_k}{\lambda_1^{Lm+k-1}\Gamma(Nm+k)}$$
$$\times\frac{1}{\ln 2}\times\left(2m+k-1\right)!\times\exp\left(\frac{1}{\lambda_1}\right)$$
$$\times\sum_{n=1}^{2m+k}\frac{\Gamma(-2m-k+n,\frac{1}{\lambda_1})}{\left(\frac{1}{\lambda_1}\right)^n} \tag{12}$$

where $\Gamma(\alpha,x)=\int_x^\infty e^{-t}t^{\alpha-1}dt$ is the upper incomplete gamma function.

IV. NUMERICAL RESULTS AND EXAMPLES

In this section the numerical results from the derived formula shown in previous section for dual-branch MRC diversity are illustrated in this section. The evaluation is implemented by the computer with the Matlab™ package software. In Fig. 2 the validation of the pdf of the output SNR obtained from the exact expression (4) with (5), and the gamma approximate expression offered in [13] for a constant correlation model with four paths, $L=4$, fading value, m=1.8, and the unit average SNR is applied, $\bar{\gamma}=1$, besides, the correlation is set as ρ = 0.64 is conducted. On the other hand, although the approximate solution matches quite well with the exact solutions in the high SNR region, it tends to deviate in the lower tail of the pdf. Hence, the approximate solution has to be used with caution as far as outage probability and average probability of errors calculations are concerned since the lower tail of the pdf is very critical for these calculations. The channel capacity per unit bandwidth of dual-branch MRC systems over correlated-Gamma fading channels as a function of average SNR, $\bar{\gamma}$, with different fading parameters, m = 1 and m = 2, respectively, and ρ = 0.49 are presented in Fig. 3. It is exact as expected the MRC diversity provides the largest improvement when the fading parameter is lager, that is, the channel capacity performance of the dual-branch MRC schemes becomes superior, and it depends on the condition which the selected propagation environment is much better. Besides, the system capacity is also evaluated with different power decay factors, μ = 0.1, 0.5, and, 0.9. It is easy to see that the channel capacity of dual-branches MRC diversity deteriorated as μ decreases. However, it is worthwhile to note that the effect of power decay factor is much less than that of the fading parameter. This is the reason that the most important factors dominate the system performance of a wireless communication system is the searching of the transmission environments. Moreover, the results shown in Fig. 4 illustrates the capacity per unit bandwidth of a dual-branch MRC system over correlated-Gamma fading channels with different values, m = 1 and m = 2, distinctive

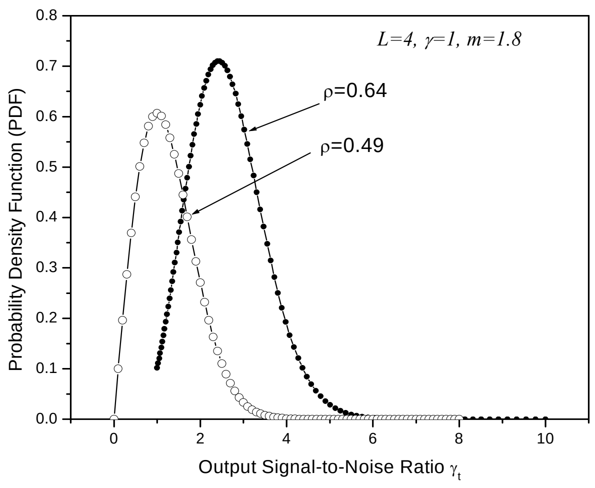

Fig. 2. The pdf plot of correlated-Gamma distributed with L=4, γ=1, m=1.8, and constant power correlation ρ=0.49, and ρ=0.64

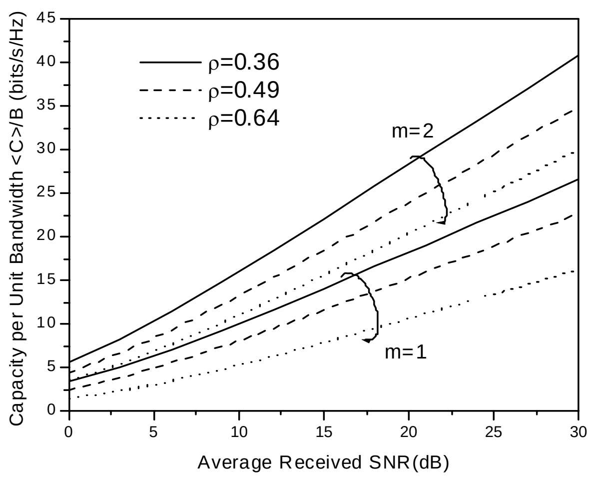

Fig. 3. The performance of channel capacity vs. average received SNR with different fading parameters and different correlation values

values of the correlation coefficients, and ρ = 0.36, 0.49, and 0.64, as a function of the average SNR, $\bar{\gamma}$. For the purpose of comparison, the channel capacity of Nakagami fading channels without diversity, which was obtained in [2, Eq. (23)], is presented in Fig. 4. It is seen that MRC diversity improves the capacity of Nakagami-*m* fading channels. It is valuable to describe that the channel capacity of the dual-branches MRC diversity is deeply degraded by the higher value of the correlation coefficient value. Furthermore, the channel capacity definitely relates to the separation between the received paths. This degradation can be resolved by increasing the distance between the receiver antennas [1].

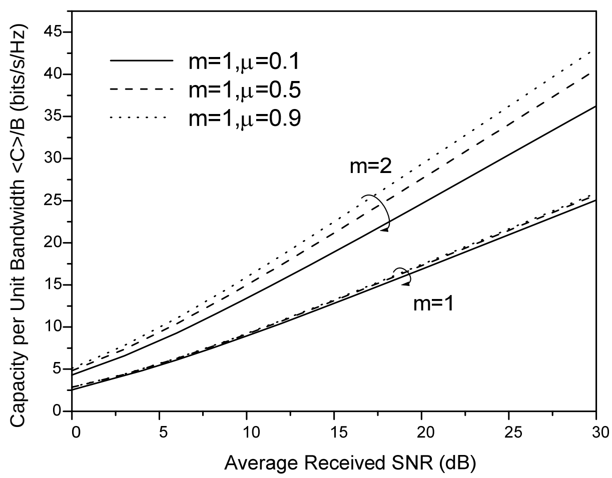

Fig. 4. Channel capacity per unit bandwidth vs. average received SNR with different fading parameters, m = 1 and m = 2, and different values of the correlation coefficient values

V. CONCLUSION

The closed-form expressions for the channel capacity of an dual-branch MRC systems over correlated-Gamma channels for integer *m* is computed in this paper. It was observed that the diversity combining techniques that are considered in this paper can provide with improvement to the capacity of Gamma-*m* fading channels. It was seen that channel capacity decreases as the correlation coefficient ρ increases. However, the decrease in capacity due to correlation diminishes as fading gets less severe. The derivative in this report relied on the Moschopoulos representation for the pdf of the sum of independent gamma variates and its extension to the pdf of the sum of correlated gamma variates to provide an complete pdf-based approach (including the parameters of fading parameter and correlation coefficients) for the capacity performance analysis of dual-branches MRC scheme over not necessarily independent nor identically distributed correlated-Gamma fading channels.

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APPENDIX A

The formula shown in (13) of the channel capacity for an dual-branch MRC schemes working in correlated-Gamma statistical model can be derived by substituting the equation (4) into (11), and obtained as

$$\eta_{MRC} = \frac{\langle C \rangle}{B} = \int_0^{\infty} \log_2(1+\gamma) P_{\gamma}(\gamma) d\gamma = \prod_{n=0}^{L} \left(\frac{\lambda_1}{\lambda_n} \right)^m \sum_{k=0}^{\infty} \frac{\delta_k U(\gamma)}{\lambda_1^{Lm+k-1} \Gamma(Nm+k)} \times \int_0^{\infty} \log_2(1+\gamma) \gamma^{Lm+k-1} e^{-\gamma/\lambda_1} d\gamma \quad \text{(A.1)}$$

where the logarithm formula, $\log_2(x)=\ln(1+x)/\ln 2$, is able to be adopted and hence the previous equation becomes as

$$\eta_{MRC} = \prod_{n=0}^{L} (\lambda_1/\lambda_n)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k-1} \Gamma(Nm+k)} \cdot U(\gamma) \times \frac{1}{\ln 2} \int_0^{\infty} \ln(1+\gamma) \gamma^{Lm+k-1} e^{-\gamma/\lambda_1} dr$$

$$= \prod_{n=0}^{L} (\lambda_1/\lambda_n)^m \sum_{k=0}^{\infty} \frac{\delta_k}{\lambda_1^{Lm+k-1} \Gamma(Nm+k)} \cdot U(\gamma) \times \frac{1}{\ln 2} \times (2m+k-1)! \times \exp(1/\lambda_1) \times \sum_{n=1}^{2m+k} \frac{\Gamma(-2m-k+n, 1/\lambda_1)}{(1/\lambda_1)^n} \quad \text{(A.2)}$$

where $U(\gamma)$ is the unit step function of γ, and the equivalent

$$\int_0^{\infty} \frac{1}{\ln 2} \ln(1+\gamma) \gamma^{2m+k-1} \exp(-\gamma/\lambda_1) d\gamma = \frac{1}{\ln 2} (2m+k-1)! \times \exp(1/\lambda_1) \sum_{n=1}^{2m+k} \frac{\Gamma(-2m-k+n, 1/\lambda_1)}{(1/\lambda_1)^n} \quad \text{(A.3)}$$

has been applied in the derivative of the previous equation [9].