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Designing a Two-Dimensional (2-D) Rake Receiver Scheme for an MC-DS-CDMA System

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ABSTRACT

On the basis of a scenario that assumes the existence of a BC (branch correlation) in the environments of an MC-DS-CDMA (multi-carrier direct-sequence coded-division multiple-access) system, a two-dimensional Rake (2-D Rake) receiver for such a system is proposed and designed in this research. The performance of the proposed system is validated by an evaluation that considers the state of the working space in frequency-selective fading environments. Furthermore, some of the system parameters, e.g., the resolvable multipath number, the finger number of a Rake receiver, the fading parameters, the power decay factor, the MIP (multipath intensity profile), and the correlation characteristics between the antennas, are adopted for analysis, thus constituting the original proposal for a scenario with this kind of receiver in this system. To validate the accuracy of the derivation, some of the numerical results are presented in this report. It is worthwhile to assert that the fading parameter of the correlated-fading model dominates the performance of the MC-DS-CDMA system and that the number of antennas also definitely affects the system performance.
***Key Words*: MC-DS-CDMA system, two-dimensional Rake (2-D Rake) receiver, MIP

二維耙式接收機於 MC-DS-CDMA 系統之研究設計

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摘　要

本文源於假設在多載波直序式分碼多重近接系統（MC-DS-CDMA system）之中，基於分支相關（BC）的背景考量，利用二維耙式（2-D Rake）接收機之設計進而改良 MC-DS-CDMA 系統的效能。隨之提出的系統改良方法，並經由效能之評估以確認系統工作於選頻性通道（frequency selective channel）中所呈現的設計之有效性。進而，透過一些系統參數加以証明，例如，多路徑解析數，耙型接收分支數，衰落參數，功率衰減因素，與多路徑強度面等等。本

文係原創提出對於 MC-DS-CDMA 系統分支相關之改善方法。值得說明的結論是；經過本研究的設計發現，不只相關衰落分支的不同模式支配著 MC-DS-CDMA 系統的效能，而且接收天線的數目也確實影響著系統效能，然而它們可以經由本文提出之方法加以改善。

關鍵詞：MC-DS-CDMA 系統，二維 Rake（2-D Rake）接收機，MIP，天線分集

I. INTRODUCTION

Recently, the wideband radio system combining with the application of multicarrier modulating and CDMA (coded-division multiple-access) schemes has been considered interesting in the cellular wireless communications. It is the reason that DS (direct-sequence) waveform exist the broader bandwidth for combating the ISI (inter-symbol interference) caused by the multipath fading during the transmission. Generally, a Rake receiver can be adopted to avoid the multipath fading and to promote the system performance whenever the bandwidth of the propagating signal exceeds the coherent bandwidth of the fading channel, which is called as the frequency selective fading channel [15, 17]. The Rake receiver provides a correlator for each carrier and the signal at the output of the correlators is combined with some tapped delay lines. The operation methods of a Rake receiver can be seen same as that of an MRC (maximal-ratio combining) diversity. As both of the multipath diversity and the antenna diversity are included into the calculation of diversity gain, the receiver model can be considered as a multiple dimension Rake receiver, *i.e.*, except the diversity gain the total received intensity of the signal can be summed up with the individual signal come from different antennas [7, 20].

However, in order to create broader bandwidth and suppress the interference effect for a wireless radio system the multicarrier scheme has been applied. Based on the motivation, the 4G (fourth generation) wireless cellular system, MC-CDMA (multi-carrier CDMA), which based on the OFDM (orthogonal frequency division multiplexing) signaling techniques, is now engaged in exploring [19]. One of the most important types of multi-carrier CDMA system is called MC-DS-CDMA (multi-carrier direct-sequence CDMA) system, which has the data sequence multiplied by a spreading sequence modulates disjoint multiple carriers. The receiver provides a correlator for each carrier and at the output of the correlators combined with MRC diversity. Generally, multi-carrier DS systems can be categorized into two types: (1) a combination of OFDM and CDMA, and (2) a parallel transmission-scheme of narrowband DS waveform in the frequency domain [8]. Furthermore, in wireless mobile communications the performance of the antenna diversity is always an attractive area of interesting to study [2, 10-11]. In additions, for the purpose of increasing the ability of the resolve and combination for multiple paths the wideband spread spectrum techniques illustrate an extra source of diversity due to the multipath phenomenon of the wireless mobile channel. Although there exist a lot of numbers of statistical distributions that well characterize the statistics of the mobile radio signal; however, empirical results as well as physical reasons have shown the fact that the Nakagami-*m* distribution is not only a more versatile model for a variety of fading environment such as urban, suburban radio multipath channels and even for indoor propagation, but also including Rayleigh distribution as a special case with the fading figure m=1 [14]. On the other hand, it is well known that the correlation phenomenon will impact the performance of the diversity combining schemes [9].

In this paper we propose a multi-dimension Rake receiver, called as 2-D Rake receiver, that adopts both multipath and antenna diversity at the output end. The system performance with BER (bit-error rate) of the MC-DS-CDMA system with 2-D Rake receiver is evaluated under the assumption both of the independent and correlated characteristics between the receiver paths. On the other hand, the performance of an MC-DS-CDMA system was analyzed over correlated-Nakagami-*m* distributed. Besides, in order to validate the accuracy of the evaluating results some of the parameters, *e. g.*, the arbitrary fading figures of fading channel, the MIP and the power delay ratio are all considered in the numerical analysis section.

The paper is organized as follows. In section II the system model of an asynchronous MC-DS-CDMA is described, i.e., we describe the transmitter, channel, and receiver models, and derive the expression for the soft decision statistics of the BPSK (binary phase shift keying) with M-D Rake receiver. A closed-form expression for the pdf (probability density function) of the decision static is shown in section III. In section IV illustrates the closed-form expression for the average BER with the impact of correlated and uncorrelated fading among spatially separated Rake fingers on system performance the numerical results are presented in section V. Finally, there is a simple conclusion drawn in section VI.

II. SYSTEM MODELS

1. Transmitter Model

In Fig. 1 the transmitter block diagram proposed in [8] was presented again for adopting as the transmitter model in this paper. An MC-DS-CDMA system in Fig. 1 where a unique spreading sequence is assumed to serve for each user and the active user employs M subcarrier, which is supposed equal to the number of the branch number of propagation channels, *i.e.*, $L=M$, and the BPSK modulation scheme is considered. The overall bandwidth of a MC-DS-CDMA system with all the subcarrier is given by $BW_M=(1+\mu)/MT_c$, where $0<\mu\leq 1$ is the roll-off factor, M is number of subcarrier, and T_c is the chip duration. From the points mentioned above, the total bandwidth of the MC-DS-CDMA system of the k-th user can be computed as $BW_T=(1+\mu)/T_c$. The transmitted signal of a MC-DS-CDMA system of the k-th user shown in Fig. 1 is given as [7]

$$s^{(k)}(\mathrm{t})=\sqrt{2E_c}\sum_{n=-\infty}^{\infty}c_n^{(k)}d_{\lfloor n/N\rfloor}^{(k)}h(t\text{-}nMT_c\text{-}\tau^{(k)})\sum_{i=1}^{\mathrm{N}}\mathrm{Re}\left[e^{j(2\pi f_i t+\theta_i^{(k)})}\right] \tag{1}$$

where E_C is the chip energy, $c_n^{(k)}$ is the pseudo-random spreading sequence, $d_{\lfloor n/N\rfloor}^{(k)}\in\{+1,-1\}$ denotes the data bit of the k-th user, $\lfloor\cdot\rfloor$ is the floor function extracts the integer value, where N indicates the length of PN (pseudo noise) sequence, $h(t)$ is the impulse response of the chip wave shaping filter, $\tau^{(k)}$ is an arbitrary time delay uniformly distributed over $[0, NMT_c]$, Re[·] denotes the real part, $\theta_i^{(k)}$ and f_i's, $i=1, 2,\ldots, M$ are a random carrier phase uniformly distributed over $(0, 2\pi]$ and the carrier frequencies, respectively.

2. Channel Model

The low-pass equivalent impulse response of the bandpass channel from the transmitter to the j-th receiver

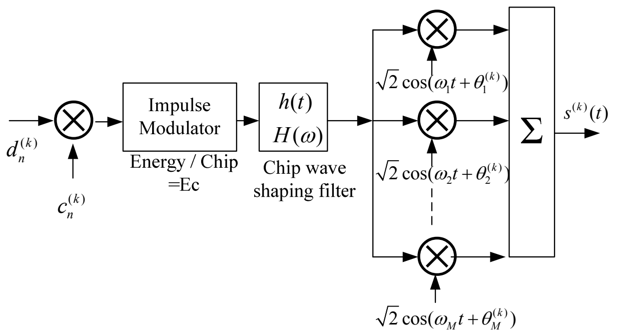

Fig. 1. The transmitter and channel block diagram of an MC-DS-CDMA system for user *k*

antenna for the k-th user can be written as

$$h_j(t)=\sum_{l=0}^{L^{(k)}-1}\alpha_{j,l}^{(k)}e^{j\varphi_{j,l}^{(k)}}\delta(\tau-\tau_{j,l}^{(k)}) \tag{2}$$

Where $L^{(k)}$ is the number of resolvable propagation paths that reach the receive antenna. Each path is characterized by its instantaneous fading amplitude $\alpha_{j,l}^{(k)}$, its phase shift, and its propagation delay $\tau_{j,l}^{(k)}$. The path phases and path delays are assumed to be independent and uniformly distributed over $[0, 2\pi]$ and $[0, T]$, respectively. A tapped delay line model describes the frequency selective channel with the l-th multipath delay of the k-th user given by $\tau_{j,l}=\tau_{j,0}+lT_c$ [3], where j is the number of antenna, is assumed in this report. Assume that the fading amplitude is modeled as Nakagami-m fading channel model, the instantaneous power of the l-th path, $l = 0, 1,\ldots, L^{(k)}-1$, can be easily shown follows the gamma pdf given as

$$f_{r_{j,l}}^{(k)}(\gamma_l)=\frac{(m_{j,l}^{(k)}/\Omega_{j,l}^{(k)})^{m_l^{(k)}}}{\Gamma(m_{j,l}^{(k)})}(\gamma_l)^{\mathrm{m}_{j,l}^{(k)}-1}\times\exp\{-\frac{m_{j,l}^{(k)}}{\Omega_{j,l}^{(k)}}\},\gamma_1\geq 0 \tag{3}$$

where $\Omega_l^{(k)}=E[(\alpha_l^{(k)})^2]$ is its average channel power. In addition, the total time average channel gain per antenna for each user is normalized to one, i.e.,

$$\sum_{l=0}^{L^{(k)}-1}E[(\alpha_{j,l}^{(k)})^2]=\sum_{l=0}^{L^{(k)}-1}\Omega_{j,l}^{(k)}=1 \tag{4}$$

In order to discuss the effect of MIP phenomena, generally it considered as exponential type and given as

$$\Omega_{j,l}^{(k)}=\Omega_{j,0}^{(k)}e^{-l\delta},\ l=0, 1,\ldots, L^{(k)}-1 \tag{5}$$

where the parameter δ indicates the rate of decay of the average path strength as a function of the path delay.

3. Receiver Model

The 2-D Rake receiver block diagram of a MC-DS-CDMA system with BPF (band-pass filter) and LPF (low pass filter) for the referenced user (the first subscriber) is illustrated in Fig. 2, in which two antennas are assigned to receive the waveform and to pass them to the corresponding band-pass filter, then the signal for the referenced user will be summed up after the Rake receiver shown in Fig. 3. The received equivalent low pass signal, $r_{LP}(t)$, can be obtained as

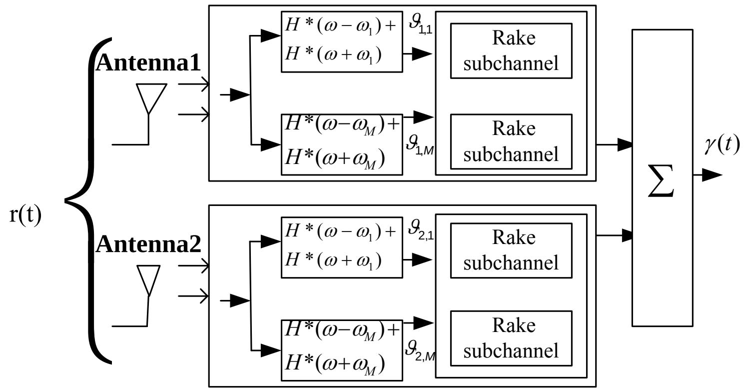

Fig. 2. The receiver block diagram of two-dimensional Rake receiver for a referenced user (first user)

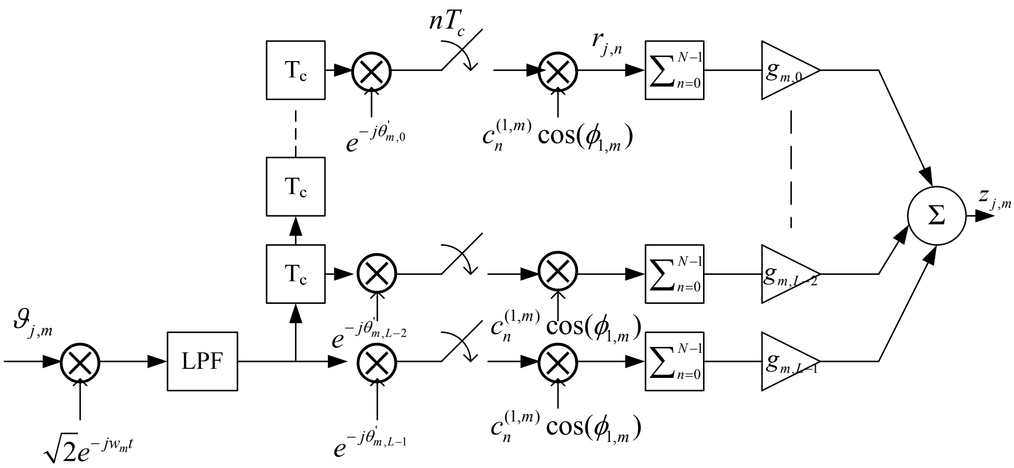

Fig. 3 The block diagram of a Rake subchannel

$$
\begin{aligned}
r_{LP}(t) &= s_1(t)\otimes c_i^{(k)}(\tau)+z(t)+z_J(t)\\
&=\left[\sum_{k=1}^{K}\sqrt{2E_c}\sum_{n=-\infty}^{\infty}d_b^{(k)}c_n^{(k)}h\left(t-nMT_c-\tau^{(k)}\right)\sum_{i=1}^{M}e^{j\theta_i^{(k)}}\right]\\
&\quad\otimes\left[\alpha_i^{(k)}e^{j\alpha_i^{(k)}}\,\delta(\tau)\right]+z(t)+z_J(t)\\
&=\sum_{k=1}^{K}\sqrt{2E_c}\sum_{n=-\infty}^{\infty}d_b^{(k)}c_n^{(k)}h\left(t-nMT_c-\tau^{(k)}\right)\cdot\sum_{i=1}^{M}\alpha_i^{(k)}e^{j\left(\theta_i^{(k)}+\alpha_i^{(k)}\right)}\\
&\quad+z(t)+z_J(t) \qquad (6)
\end{aligned}
$$

where the subscript LP of $r_{LP}(t)$ represents the activity of low-pass, the symbol ⊗ denotes the convolution operator, $\delta(\tau)$ is the impulse function, $z(t)$ is the equivalent of low pass AWGN, $z_J(t)$ indicates PBI after low pass filter. The complex low-pass equivalent impulse response of the *i-th* channel is $\{c_i=\xi_i\cdot\delta(t),\ i=1,\ldots,\ M\}$, and $\xi_i^{(k)}=\alpha_i^{(k)}\exp(j\varepsilon_i^{(k)})$, where $\alpha_i^{(k)}$ and $\varepsilon_i^{(k)}$ are corresponding to represent the attenuation factor and phase-shift of the i-th channel of the k-th user. The complex equivalent impulse response of the channel is expressed as $c_I(t)=\sum_{l=1}^{L}\alpha_l\delta(t\text{-}lT_c)$. Hence the received signal at the output of receiver is given as [8]

$$
\begin{aligned}
r(t) = \sum_{k=1}^{K}\Bigg\{&\sqrt{2E_c}\sum_{n=-\infty}^{\infty}d_h^{(k)}c_n^{(k)}h(t-nMT_c-\tau^{(k)})\\
&\times\sum_{i=1}^{M}\alpha_i^{(k)}\cos(2\pi f_i t+\psi_i^{(k)})\Bigg\}+N_w(t)+N_J(t) \qquad (7)
\end{aligned}
$$

where K denotes the user number, $\psi_i^{(k)}=\theta_i^{(k)}+\varepsilon_i^{(k)}$, $N_w(t)$ is AWGN term with a double-sided PSD (power spectral density) of $\eta_0/2$, $N_J(t)$ is partial band of Gaussian interference with a PSD of $S_{n_j}(f)$, which is written as

$$
S_{n_J}(f)=\begin{cases}\dfrac{\eta_J}{2},\ f_J-\dfrac{W_J}{2}\le|f|\le f_J+\dfrac{W_J}{2}\\ 0,\ otherwise\end{cases} \qquad (8)
$$

where f_J and W_J represent the bandwidth of the interference and the center frequency, respectively. Then the interference (Jamming)-to-signal ratio, JSR, is defined as the ratio of the interference power value to signal power value, and can be written as

$$\mathrm{JSR} = \frac{\eta_J W_J}{E_b / T} = (1+\mu)\frac{\eta_J}{E_b}\frac{N}{M} \tag{9}$$

The output signals after LPF, $\zeta_i^{(1)}(t)$, i=1,…, M, of the chip-matched filter in the branch for the referenced user is given by

$$\begin{aligned}
&\zeta_i^{(1)}(t) \\
&= Lp\left\{\left[r(t)\otimes \Im^{-1}\left\{H^*(\omega - \omega_i)+H^*(\omega+\omega_i)\right\}\right]\cdot\sqrt{2}\,cos\left(\omega_i t+\theta_i^{\prime(1)}\right)\right\} \\
&= \sum_{k=1}^{K}\sum_{n=-\infty}^{\infty}\sqrt{E_c}\,d_b^{(k)}c_n^{(k)}\alpha_i^{(k)}\cos\left(\theta_i^{r(k)}-\theta_i^{\prime(1)}\right)\Im^{-1}\left\{|\,H(\omega)|^2\, e^{-j\omega(nMT_c+\tau_k)}\right\} \\
&\quad + Lp\left\{\left[n(t)\otimes \Im^{-1}\left\{H^*(\omega - \omega_i)+H^*(\omega+\omega_i)\right\}\right]\cdot\sqrt{2}\,cos\left(\omega_i t+\theta_i^{\prime(1)}\right)\right\} \\
&\quad + Lp\left\{\left[n_J(t)\otimes \Im^{-1}\left\{H^*(\omega - \omega_i)+H^*(\omega+\omega_i)\right\}\right]\cdot\sqrt{2}\,cos\left(\omega_i t+\theta_i^{\prime(1)}\right)\right\} \\
&= D_{\zeta_i}^{(1)}(t)+MAI_{\zeta_i}^{(1)}(t)+JSR_{\zeta_i}^{(1)}(t)+N_{\zeta_i}^{(1)}(t)
\end{aligned} \tag{10}$$

where the symbol $Lp\{\cdot\}$ is applied to express the function of LPF, $\Im^{-1}\{\cdot\}$ represents the inverse Fourier transform, $H^*(\omega - \omega_i)+H^*(\omega+\omega_i)$ is the frequency response of the BPF in the receiver of MC-DS-CDMA system. The first term of (10) indicates the desired signal of the referenced user can be written as

$$D_{\zeta_i}^{(1)}(t) = \sqrt{E_c}\,\alpha_i \sum_{n=-\infty}^{\infty} d_n c_n x(t-nMT_c) \tag{11}$$

where α_i, i=1,…, M, denotes as fading envelopes characterized by Nakagami-m RV's (random variables). The second term in (10) is the interference comes from the other users, called as MAI (multiple access interference), and can be determined as (the superscript of user will be omitted form here)

$$MAI_{\zeta_i}(t) = \sum_{k=2}^{K}\left\{\sqrt{E_c}\,\xi_i \sum_{n=-\infty}^{\infty} d_n\cdot c_n\cdot x(t-nMT_c - \tau)\right\} \tag{12}$$

where $\xi i \equiv \alpha_i \cos\phi_i$ is the envelope of the MAI component, which is allowed to approximate Gaussian random variable under the assumption with large user number K [19]. The third term in (10) is the JSR defined in (9) and can be represented as

$$JSR_{\zeta_i}(t) = Lp\left\{\sqrt{2}\,n_{i,j}(t)\cos(2\pi f_i t+\psi_i)\right\} \tag{13}$$

and the last term of (10) presents the output signal caused by the fact that the AWGN passes through the LPF, and it can be expressed as

$$N_{\zeta_i}(t) = Lp\left\{\sqrt{2}\,n_{i,w}(t)\cos(2\pi f_i t+\psi_i)\right\} \tag{14}$$

where the terms $n_{i,j}(t)$ in (13) and $n_{i,w}(t)$ in (14) are results from passing $N_J(t)$ and $N_w(t)$ into (7), respectively, through the i-th bandpass filter. It is necessary to evaluate the SNR (signal-to-noise ratio) at the output of the receiver for the referenced user such that the system performance can be achieved. Thus all of the statistical characteristics of the signal at the output of the i-th correlator are to be determined and expressed as

$$\chi_i = D_{\chi_i} + MAI_{\chi_i} + JSR_{\chi_i} + N_{\chi_i} \tag{15}$$

where terms shown in previous equation are adopted as the same results that evaluated and shown in [8].

The concatenated orthogonal/PN spreading sequences can be considered as random signature sequences. Although there are some approaches that do not rely on Gaussian approximations for the MAI of random sequences [13], in this work, we follow up the analysis in [13] and [4], in which employs the SGA (standard Gaussian assumption) to approximate the interference induced by the other users on the desired user. Therefore, conditioned on the channel amplitudes $\alpha_{j,n}$, the noise variance of the n-th Rake finger of the j-th antenna due to all the multiple access users in the same cell is given by

$$\sigma_{mai,j,n}^2 = \frac{E_b T}{6N}(\alpha_n)^2\sum_{k=2}^{K}\sum_{l=0}^{L-1}\Omega_{j,l} \tag{16}$$

Similarly, the variance of the SI (self-interference) due to multipath is approximated by [13]

$$V_{ar}\{J_{Zi}\} = NR_{j_i}(0)+\sum_{a=1}^{N-1}R_{j_i}(aMT_c)\cdot 2\sum_{n=a}^{N-1}c_n c_{n-a} \tag{17}$$

and the variance due to AWGN is

$$V_{ar}\{N_{Zi}\} = N\frac{\eta_0}{2} \tag{18}$$

where $\Omega_{j,l}$ denotes the average power of the l-th path in the j-th antenna, and E_b=PT is the received signal energy per bit per antenna. Without loss of generality, we assume identical MIP among receive antennas (*i.e.*, $\Omega_{j,l}$ =Ω_l, j=1,2,…,M_r). It follows that the desired signal of the 2-D Rake receiver (conditioned on the path amplitude, α_n) is a Gaussian random process with mean

$$U_s = \sqrt{\frac{E_b T}{2}} \sum_{j=1}^{2} \sum_{n=0}^{L_r-1} (\alpha_{j,n})^2 \tag{19}$$

and the variance is given by the total, σ_T^2, interference term with the expression as

$$\sigma_T^2 = Var\{MAI_{Zi}\} + Var\{N_{Zi}\} + Var\{J_{Zi}\}$$
$$= \frac{(\mathrm{K}-1)NE_c\Omega_i}{2}\left(1-\frac{\mu}{4}\right) + \frac{N\eta_0}{2} + \frac{N\eta_J}{4} \tag{20}$$

Therefore, for the 2-D Rake CDMA receiver, the SINR (signal-to-interference-and-noise ratio) at the output of an MRC output is written as

$$\gamma = \frac{U_s^2}{2\sigma_T^2} = \sigma_0 \sum_{j=1}^{2} \sum_{n=0}^{L_r-1} (\alpha_{j,n})^2 \tag{21}$$

Furthermore, with the help of (4), the term σ_0 in the previous equation can be obtained as

$$\sigma_0 = \left\{ \frac{m(K-1)}{N}(1-\frac{\mu}{4}) + \frac{1}{\eta_0 / E_b} mL \right\}^{-1} \tag{22}$$

III. STATISTICAL ANALYSIS OF 2-D RAKE DECISION

The instantaneous SINR γ at the output of the 2-D Rake receiver was shown in the previous section, and the expression can be written as

$$\gamma = \sum_{j=1}^{2} \sum_{n=0}^{L_r-1} \gamma_{j,n} \tag{23}$$

where the antenna number at the receiver is preset as M_r=2 for the two-dimensional Rake receiver, L_r indicates the finger number of the Rake receiver, and $\gamma_{j,n}$=$\sigma_0(\alpha_{j,n})^2$ is the instantaneous SINR of the n-th finger of he j-th antenna. For the purpose of evaluating the system performance of the dual-Rake receiver proposed in this report. The computation of statistical characteristics of the SINR is required, i.e., the computation of pdf of SINR is necessary. It is known that the expression of the pdf of γ modeled as Nakagami-m fading with the integer value of fading figure (fading parameter) has been illustrated in [1]. In words, when the fading figure are real and arbitrary, the pdf of γ is given as an approximate expression and shown in [14], it can be expressed as an indefinite integral in [15] too. The other most common mean for determining the pdf of γ is the method of adopting the product of the MGF (moment generating function), and followed by the calculation of the inverse Laplace transform. Hence, by means of substituting (21) into the MGF formula the MGF of $\gamma_{j,n}$ can be expressed as

$$M_{\gamma_{j,n}}(t) = \int_0^\infty e^{-xt} f_{\gamma_{j,n}}(x)dx$$
$$= \frac{(y_{j,n})^{m_{j,n}}}{\Gamma(m_{j,n})} \int_0^\infty e^{-xt} x^{m_{j,n}-1} \exp(-y_{j,n}x)dx \tag{24}$$

Where

$$y_{j,n} = \frac{m_{j,n}}{\sigma_0 \Omega_{j,n}}, \; j=1,2,\ldots, M_r,\; n=0,1,\ldots,L_r\text{-}1 \tag{25}$$

By adopting the closed form defined in [6], the formula (24) can be computed by some of the steps as follows. Based on some of the integral formulas, shown in [11] and [16], the closed form of integral in (24) can be easily evaluated. However, the integral in (24) will be determined by using of the approach proposed in [1]. Firstly, the exponential function may be expressed as a contour integral $\exp(-x) = \int_{-i\infty}^{i\infty} \Gamma(-s)x^s ds / 2\pi i$ [15], where $i=\sqrt{-1}$. By interchanging the order of integration and substituting it into the MGF formula (24), which can be derived as

$$M_{\gamma_{j,n}}(t)$$
$$= \frac{(y_{j,n})^{m_{j,n}}}{\Gamma(m_{j,n})} \frac{1}{2\pi i} \times \int_{\mathrm{C}} \Gamma(-s)(y_{j,n})^s \int_0^\infty x^{m_{j,n}+s-1} e^{-xt} dxds$$
$$= \frac{1}{\Gamma(m_{j,n})} (\frac{y_{j,n}}{t})^{m_{j,n}} \frac{1}{2\pi i} \times \int_{\mathrm{C}} \Gamma(-s)\Gamma(m_{j,n}+s)(\frac{y_{j,n}}{t})^s ds \tag{26}$$

where the components of the multipath channel are assumed all identical and via the imaginary axis (in the complex s-plane), separating the poles of $\Gamma(m_{j,n}+s)$, j=1,2,…, M_r, n=0,1,…, L_r-1 from the poles of $\Gamma(-s)$. Thus following the inverse Laplace then the pdf, $f_\gamma(\gamma)$, of the SINR can be calculated in terms of the confluent form of the multivariate Lauricella hyper-geometric function and obtained as [18]

$$f_\gamma(\gamma)=\frac{1}{\Gamma(\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n})}\left[\prod_{j=1}^{2}\prod_{n=0}^{L_r-1}(y_{j,n})^{m_{j,n}}\right]$$
$$\times\gamma^{\left(\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n}\right)-1}\Phi_2^{(2gL_r)}(m_{1,0},m_{1,1},\dots m_{2,L_r-1},$$
$$\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n},-y_{1,0}\gamma,,-y_{1,1}\gamma,\dots,-y_{2,L_r-1}\gamma) \quad (27)$$

where $\Phi_2^{(n)}(b_1,\dots b_n;c,x_1,\dots,x_n)$ is the confluent Lauricella hyper-geometric function define as [18]

$$\Phi_2^{(n)}(b_1,\dots b_n;c,x_1,\dots,x_n)=\sum_{i_1,\dots,i_n=0}^{\infty}\frac{(b_1)_{i_1}\dots(b_n)_{i_n}}{(c)_{i_1+\dots+i_n}}\frac{x_1^{i_1}}{i_1!}\dots\frac{x_n^{i_n}}{i_n!} \quad (28)$$

In (28) the parameters $y_{j,n}$ are defined in (25) equals to the ratio of the amount of fading $m_{j,n}$ divided by the corresponding average SINR, $\sigma_0\Omega_{j,n}$, of the n-th Rake finger of the j-th antenna. For the negative exponential MIP with power decay factor, δ, from (4) and (5) , the average power of the n-th path can be written as [2]

$$\Omega_{j,n}=\frac{e^{-n\delta}}{q(L,\delta)},\ j=1,2,\dots,M_r,\ n=0,1,\dots,L_r\text{-}1 \quad (29)$$

where the number of multipath L is considered for the desired user and the

$$q(L,\delta)=\sum_{l=0}^{L-1}e^{-l\delta}=\frac{1-e^{-L\delta}}{1-e^{-\delta}} \quad (30)$$

IV. BIT ERROR PROBABILITY

1. The Effect of Un-Correlated Channels

The coherent BPSK (binary phase shift keying) was applied as the modulator at the post-detector. It is known that the conditional BER of that in AWGN channel is given by [6, 15]

$$P_E(\gamma)=Q(\sqrt{2\gamma})=\frac{\Gamma(\frac{1}{2},\gamma)}{2\sqrt{\pi}} \quad (31)$$

where $Q(\cdot)$ is the Marcum-Q-function, and $\Gamma(\alpha,x)=\int_x^{\infty}t^{\alpha-1}e^{-t}dt$ denotes the complementary incomplete gamma function [17, (8.350-2)]. Therefore, the system BER is determined as an equation function of Lauricella multivariate hyper-geometric and shown as (see Appendix A)

$$\overline{P}_E=\frac{\Gamma(\frac{1}{2}+\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n})}{2\sqrt{\pi}\Gamma(1+\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n})}\left[\prod_{j=1}^{2}\prod_{n=0}^{L_r-1}(\frac{y_{j,n}}{y_{j,n}+1})^{m_{j,n}}\right]$$
$$\times F_D^{(2\cdot L_r)}(\frac{1}{2}+m_{1,0},m_{1,1},\dots,m_{2,L_r-1};1+\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n},$$
$$\frac{y_{1,0}}{y_{1,0}+1},\frac{y_{1,1}}{y_{1,1}+1},\dots,\frac{y_{2,L_r-1}}{y_{2,L_r-1}+1}) \quad (32)$$

The average BER presented in the previous equation converges for all practical values of the system parameters (fading parameters), and where the representation in (32) provides a convenient method for fast and accurate numerical computation of the multivatiate hyper-geometric function.

2. The Effect of Correlated Channels

Although multipath fading can be assumed to be independent because paths with different delays arrive at each antenna after traveling different routes, each path with the same delay may suffer correlated fading among the spatially separated receive antennas. The degree of correlation depends on the distance between the antennas and their configurations [2, 10].

Similarly, the average BER in the spatially correlated Nakagami-m fading channel becomes (see Appendix B)

$$\overline{P}_E=\frac{\Gamma(\frac{1}{2}+\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n})}{2\sqrt{\pi}\Gamma(1+\sum_{j=1}^{2}\sum_{n=0}^{L_r-1}m_{j,n})}\left[\prod_{j=1}^{2}\prod_{n=0}^{L_r-1}(\frac{y_{j,n}}{y_{j,n}+\lambda_j^{(n)}})^{m_n}\right]$$
$$\times F_D^{(2\cdot L_r)}(\frac{1}{2}+m_{1,0},m_{1,1},\dots,m_{M_r,L_r-1};1+2\sum_{n=0}^{L_r-1}m_n;$$
$$\frac{y_{1,0}}{y_{1,0}+\lambda_j^{(0)}}\gamma_{th},\frac{y_{1,1}}{y_{1,1}+\lambda_1^{(1)}}\gamma_{th},\dots,\frac{y_{2,L_r-1}}{y_{2,L_r-1}+\lambda_2^{(L_r-1)}}\gamma_{th}) \quad (33)$$

V. NUMERICAL RESULTS AND DISCUSSION

In this section the effects of system and channel parameters on the average BER of a coherent BPSK 2-D Rake receiver of an asynchronous MC-CDMA system working in both un-correlated and correlated Nakagami-m fading environment with arbitrary fading parameters are evaluated numerically. Accordingly, the un-correlated fading effects are presented with the formula (32) and the correlated fading effects are illustrated in the equation (33). The effect of the power decay factor, δ, with different antenna numbers, M_r=1 (1D-Rake) and M_r=2 (2D-Rake), and different Rake finger numbers, L_r=2, 4 and 6, are presented in Fig. 4 and Fig. 5 respectively. It is except reasonable to describe that the system performance become more superior when the antenna number increase and the lever number of power decay factor will deteriorate the system performance. The fading MIP and the degradation of the number of Rake finger on the system BER performance is shown in Fig. 5, in which the finger number of Rake receiver are varied with L_r = 2, 4 and 6, the subscriber K=25, and power decay factor are varied with δ= 0, 0.5, and 1. It is valuable to describe from this plot that the system performance can be improved by adding the extra number of the Rake receiver. Moreover, consider the much more of the fading pathos, *i. e.*, 6 paths, it is known that the higher values of power decay factor can supply better performance when the finger number of Rake is L_r =2. The lower decay factor includes the higher power. In words, when the Rake finger is 4, the medium values of power decay factor can supply better performance inversely compared to the lower values. The reasons are that: (1) The balance of the combined Rake fingers can be reached, (2) The power of the combination of the Rake receiver is stronger, and thus it can obtain higher gain at the diversity receiver. The other thing worth while noting is the lower value of power decay; the better performance is when all the fading pathos equal to the finger number of Rake receiver, *i.e.* $L = L_r = 6$. This feature is same as that of being described in Fig. 6.

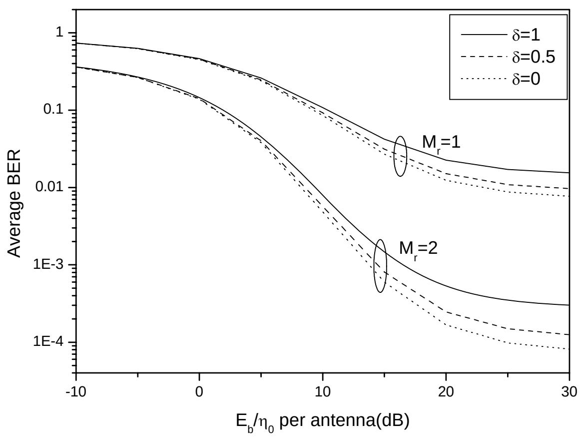

Fig. 4. Average BER versus E_b/η_0 per antenna for 1-D and 2-D Rake receiver in Rayleigh fading *with L = L_r = 4 and K* = 25

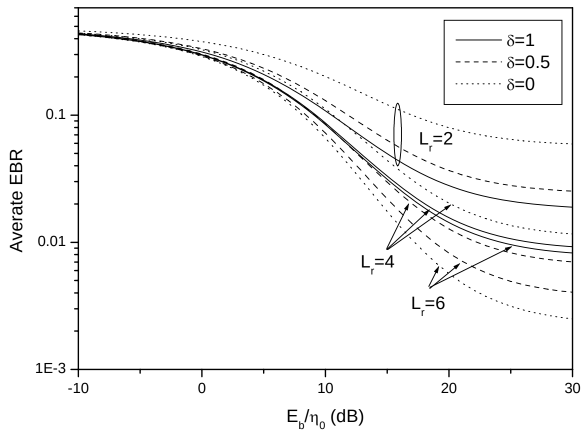

Fig. 5. Average BER versus E_b/η_0 for 1-D Rake receivers in Rayleigh fading with L = 6 and K=25

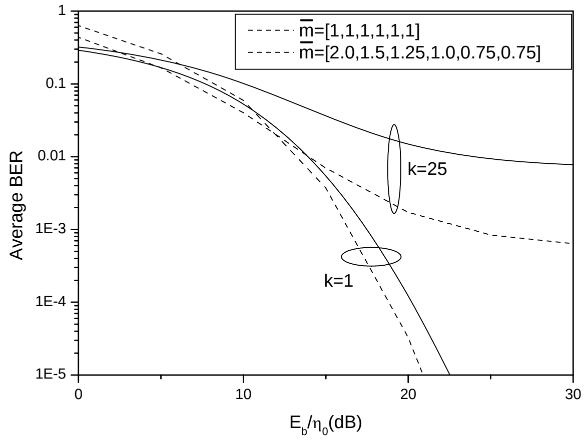

Fig. 6. Average BER versus E_b/η_0 per antenna for 1-D Rake receivers in Nakagami-*m* fading with $L = L_r = 6$ and MIP decay factor δ = 0.5

In Fig. 6 the different user numbers, K=1, and 25 are presented with the results of corresponding to the distinctive, $\overline{m}$ = [1, 1, 1, 1, 1, 1] and $\overline{m}$ = [2.0, 1.5, 1.25, 1.0, 0.75, 0.75]. The fading finger and the decay factor δ are set as $L = L_r = 6$ and δ = 0.5, respectively. It is clear to say that the system performance will be definitely degraded by the subscribers since the reason of MAI phenomena. The bit SNR is fixed in E_b/η_0=15dB per antenna, different antenna number, M_r = 1 and M_r = 2, and different decay factor, δ = 0, 0.5 and 1, are conduct for comparison with the channel capacity performance. Show in Fig. 7. The results shown in Fig. 7 described the same outcomes that shown in Fig. 4. The results presented in Fig. 8 mentioned about the effect of channel correlation in which the coefficients are set as ρ = 0 and 0.7, and the fading parameter is fixed in $\overline{m}$ = [1.5, 1.25, 1.0, 0.75] with 4 fading paths.

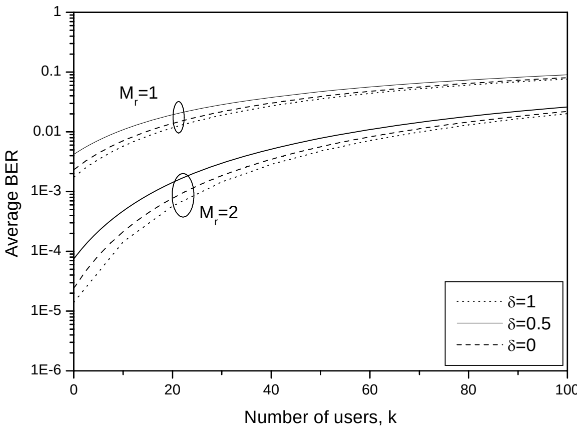

Fig. 7. Average BER versus number of users *K* for 1-D and 2-D Rake receivers in Rayleigh fading with $L = L_r$ = 4 and E_b/η_0 =15dB per antenna

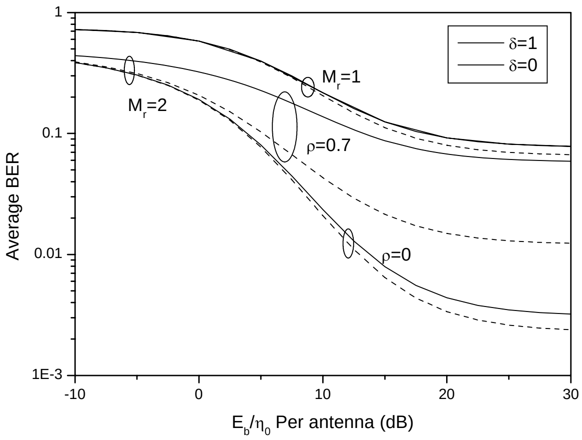

Fig. 8. Average BER versus E_b/η_0 per antenna for 1-D and 2-D Rake receiver in Nakagami-*m* fading with $L = L_r$ = 4, $\overline{m} = [1.5, 1.25, 1.0, 0.75]$ **per antenna, and *K*=50**

The inferior system performance is. Especially, the distance between different decay factors, $\delta = 0$ and $\delta = 1$, will become much larger as the correlation coefficient increases. In Fig. 9 the bit SNR versus BER curves are illustrated for comparing with the different number of subcarrier number. There are three different subcarrier umbers, N = 128, 256 and 512, are illustrated, the path number and the subscriber number are $L = L_r = 4$ and k = 50, respectively. It is worthwhile noting that the larger number of subcarrier the superior performance of the MC-CDMA system is. This is accordance with the research reports in [8].

VI. CONCLUSIONS

The extension of system performance of MC-DS-CDMA system is investigated in this paper. The antenna diversity is

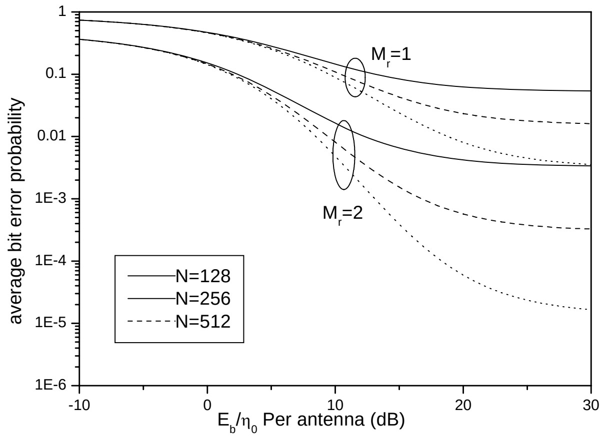

Fig. 9. Average BER versus E_b/η_0 per antenna for 1-D and 2-D Rake receivers in Rayleigh fading with $L = L_r$ = 4 and *K*=50

adopted as a two-dimensional Rake receiver and the fading channel is characterized as the versatile Nakagami-*m* statistical distributed. The most important parameters are applied for the numerical analysis, *e.g.*, the fading path number, the subscriber number, the power decay factor, the correlation of fading branches, and the fading *m*-value etc.. It is reasonable for the analysis in this paper compared with the previous researched report. However, the interesting thing needed to claim is that the dimension of the antennal definitely affects the system performance of the MC-DS-CDMA systems.

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APPENDIX A

By means of the random process the average BER of the dual-dimension Rake for MC-CDMA system working in Nakagami-m fading channel is then written as

$$\overline{P}_E = \int_0^\infty P_E(\gamma) f_\gamma(\gamma) d\gamma \tag{A-1}$$

It is known that the definition of the Lauricella multivariate hyper-geometric function $F_D^{(n)}(...)$ shown as [19, 20]

$$F_D^{(n)}(\alpha, b_1,...,b_n; c; x_1,...,x_n) = \sum_{i_1...i_n=0}^{\infty} \frac{(\alpha)_{i_1+...+i_n} (b_1)_{i_1},...,(b_n)_{i_n}}{(\mathrm{c})_{i_1+...+i_n}} \frac{x_1^{i_1}}{i_1!} ... \frac{x_1^{i_n}}{i_n!}, \ |x_1|<1,\dots, |x_n|<1$$

$$= \frac{\Gamma(c)}{\Gamma(\alpha)\,\Gamma(c-\alpha)} \int_0^1 t^{\alpha-1} (1\text{-}t)^{c-\alpha-1} \prod_{i=1}^{n} (1 - x_i t)^{-b_i} dt, \ \mathrm{Re}(c) > \mathrm{Re}(\alpha) > 0 \tag{A-2}$$

where the Lauricella function $F_D^{(n)}(...)$ for the order of n =2 is able to provided with a library function in a common mathematical software package. By using of the following transformation and the convergence of the Lauricella multivariate hyper-geometric function in (A-2) can be derived advanced [5]

$$F_D^{(n)}(\alpha, b_1,...,b_n; c; x_1,...,x_n) = \left[\prod_{i=1}^{n} (1-x_i)^{-b_i}\right] = F_D^{(n)}\left(c-a, b_1,..., b_n; c; \frac{x_1}{x_1-1},..., \frac{x_n}{x_n-1}\right) \tag{A-3}$$

Hence, the system BER can be evaluated as an equation function of Lauricella multivariate hyper-geometric as

$$\overline{P}_E = \frac{\Gamma(\frac{1}{2} + \sum_{j=1}^{2} \sum_{n=0}^{L_r-1} m_{j,n})}{2\sqrt{\pi}\,\Gamma(1 + \sum_{j=1}^{2} \sum_{n=0}^{L_r-1} m_{j,n})} \left[\prod_{j=1}^{2} \prod_{n=0}^{L_r-1} (\frac{y_{j,n}}{y_{j,n}+1})^{m_{j,n}}\right] \times F_D^{(2 \cdot L_r)}(\frac{1}{2} + m_{1,0}, m_{1,1},..., m_{2,L_r-1}; 1 + \sum_{j=1}^{2} \sum_{n=0}^{L_r-1} m_{j,n},$$

$$\frac{y_{1,0}}{y_{1,0}+1}, \frac{y_{1,1}}{y_{1,1}+1},..., \frac{y_{2,L_R-1}}{y_{2,L_R-1}+1}) \tag{A-4}$$

APPENDIX B

Let the sequence of branch SINRs, $\{\gamma_j\}_{j=1}^{M_r}$, be a set of correlated, but not necessarily identically distributed, gamma varieties with parameters m_n and $\sigma_0\Omega_{j,n}$, respectively, and let $\rho_{i,j}^{(n)}$ denote the correlation coefficient between $\gamma_i^{(n)}$ and $\gamma_j^{(n)}$, i.e.

$$\rho_{i,j}^{(n)} = \rho_{j,i}^{(n)} = \frac{Cov(\gamma_i^{(n)},\gamma_j^{(n)})}{\sqrt{\text{var}(\gamma_i^{(n)} \cdot \text{var}(\gamma_j^{(n)})}}, 0 \le \rho_{i,j}^{(n)} \le 1 \tag{B-1}$$

where i, j =1,2,…, M_r, and n=0,1,…,L_r-1. From [12], we find that the CF (characteristic function) of the instantaneous SINR γ given in (23) is obtained as

$$\phi_\gamma(t) = \prod_{n=0}^{L_r-1} \left| I_{m_r} + tA^{(n)}C^{(n)} \right|^{-m_m} = \prod_{n=1}^{L_r-1} \prod_{j=1}^{2} (1 + t \cdot (y_{j,n})^{-1} \lambda_j^{(n)})^{-m_n} \tag{B-2}$$

where $|\cdot|$ is the determinant operator, and I_{M_r} is the $M_r \times M_r$ identity matrix. The matrices $A^{(n)}$, n=0,1,…,L_r-1, are $M_r \times M_r$ diagonal matrices with entries $\sigma_0 \Omega_{j,n} / m_n = (y_{i,n})^{-1}$, and $C^{(n)}$, n=0,1,…,L_r-1, are $M_r \times M_r$ positive definite matrices defined by

$$C^{(n)} = \begin{bmatrix} 1 & \sqrt{\rho_{12}^{(n)}} & \cdots & \sqrt{\rho_{12}^{(n)}} \\ \sqrt{\rho_{21}^{(n)}} & 1 & \cdots & \sqrt{\rho_{22}^{(n)}} \\ \text{M} & \cdots & \cdots & \text{M} \\ \sqrt{\rho_{21}^{(n)}} & \cdots & \cdots & 1 \end{bmatrix} \tag{B-3}$$

In (B-2), $\lambda_i^{(n)}$, j =1, 2,…, M_r, are the M_r eigenvalues of matrix $C^{(n)}$. In the case of independent fading among the receive antenna, we have $\lambda_i^{(n)} = 1$, j =1, 2,…, M_r, n=0,1,…,L_r-1. It can be shown that (B-2) reduces to [18, p. 44].

$$\phi_\gamma(t) = \prod_{n=0}^{L_r-1} \prod_{j=1}^{2} ((y_{j,n})^{-1} \lambda_j^{(n)} t)^{-m_n} {}_1F_0(m_n; -; -\frac{y_{j,n}}{\lambda_j^{(n)} t}) \tag{B-4}$$

The confluent hyper-geometric function in (B-4) may be written as a Barnes-Mellin contour-type integral [18, p. 43]:

$${}_1F_0(m_n; -; -p \cdot t) = \frac{1}{2\pi i} \int \frac{\Gamma(m+s)}{\Gamma(m)} \Gamma(-s(p \cdot t)^s ds \tag{B-5}$$

By substituting (B-4) into (B-5), the CF becomes as

$$\phi_\gamma(t) = (\frac{1}{2\pi i})^{2L_r} \int_{C_1} \int_{C_2} \cdots \int_{C_{2L_R}} \left\{ \prod_{j=1}^{2} \prod_{n=0}^{L_r-1} \frac{1}{\Gamma(m_n)} \left(\frac{y_{j,n}}{\lambda_j^{(n)} t} \right)^{m_n} \times \Gamma(-s_{j,n}) \Gamma \Gamma m_m + s_{j,n}) \left(\frac{a_{j,n}}{\lambda_j^{(n)} t} \right)^{s_{j,n}} \right\} ds_{1,0} ds_{1,1} \ldots ds_{2,L_t-1}$$

(B-6),

which is very similar to (28). It then follows that the pdf of γ in the case of correlated fading among Rake fingers with the same path delay in spatially separated antennas is given by

$$f_{\gamma}(\gamma)=\frac{1}{\Gamma(2\cdot\sum_{n=0}^{L_r-1}m_n)}\left[\prod_{j=1}^{2}\prod_{n=0}^{L_r-1}\left(\frac{y_{j,n}}{\lambda_j^{(n)}t}\right)^{m_n}\right]\gamma^{(2\cdot\sum_{n=0}^{L_r-1}m_n)-1}\times\Phi_2^{2,L_r}(m_{1,0},m_{1,1},\ldots,m_{2,L_r-1};2\sum_{n=0}^{L_r-1}m_n$$

$$-\frac{y_{1,0}}{\lambda_1^{(0)}}\gamma,-\frac{y_{1,1}}{\lambda_1^{(1)}}\gamma,\ldots,-\frac{y_{2,L_r-1}}{\lambda_2^{(L_r-1)}}\gamma) \quad \text{(B-7)}$$

with the only restriction that $m_{j,n}=m_n$ for $j=1, 2,\ldots, M_r$. By comparing (B-7) with (33) for the independent fading case, we can obtain the outage and average BER expressions for the correlated fading case by replacing $y_{j,n}$ with ($y_{j,n}/\lambda_j^{(n)}$).