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Control of Wave Propagation in Periodic Structures Having Defects

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ABSTRACT

In this study, the influence on wave propagation of periodic structures having defects is investigated. Defects are used in periodic structures to study the unique characteristics of wave propagation in a periodic system. A structural rod model is developed to study the problems of one-dimensional periodic structures with defects.

A finite element model and a transfer matrix method are developed to study these problems to predict the performance of the system; moreover, a shape memory alloy (SMA) is used to control the characteristics of the system. The behaviors of the periodic structure are evaluated at different length ratios and material properties of the defects. The effects of the SMA when applied at various temperatures are also discussed. The location and width of stop bands can be changed with different configurations of the additional defects.

***Key Words*: periodic structure, defect, SMA, stop band**

具缺陷週期結構之波傳主動控制分析

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摘 要

本文研究具缺陷週期結構之波傳特性，利用有限元素法與轉移矩陣法，相互配合分析，對於週期性含缺陷結構進行波傳特性分析。首先比較完美的週期結構，與含缺陷的週期結構，兩者之間頻溝變化，接下來改變缺陷之幾何參數、材料參數，觀察系統之頻溝的改變。由於週期結構裡的形狀記憶合金會隨著溫度的改變而改變它的材料參數，所以本文也針對形狀記憶合金的特性，改變溫度去探討觀察頻溝的變化。希望藉由本文的分析，控制缺陷的幾何、材料參數、溫度，可發展出比單純週期結構還要更優越的濾波能力。

關鍵詞：週期結構，缺陷，形狀記憶合金，停止能帶

I. INTRODUCTION

Earlier works about the periodic structures were presented by Brillouin [2]. The propagation of waves in periodic structures formed by the assemblages of beams and plates was investigated by Heckl [3]. Later, Mead and Wilby [8] considered the effects of the damping of the multi-supported beam structures. Furthermore, Mead [6, 7] also presented the outstanding contributions to the study of the dynamics of periodic structures.

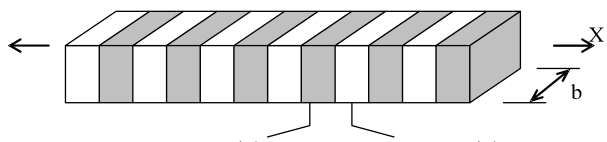
Recently, many smart materials are used to control the wave propagation of periodic structures. Ruzzene and Baz [9] studied the control of wave propagation in periodic composite rods with shape memory alloy inserts. They [10] also discussed the attenuation and localization of the wave propagation in periodic rods with SMA insert. The viscoelastic materials exhibited both viscous and elastic characteristics and the modulus of a viscoelastic material was usually represented as a complex modulus.

For periodic structures, another approach, the transfer matrix method, was used to investigate the dynamics for wave propagation in many media. Lin and Mcdaniel [5] were pioneering in the application of a transfer matrix to the analysis of stiffened plate vibration and periodic structures. Ruzzene and Baz [11] also used the method of the transfer matrix to predict pass and stop frequency bands for different proportional control gains. Then, Solaroli *et al.* [12] investigated the characteristics of wave propagation on the periodic stiffened shell by the finite element method base on the transfer matrix. Benaroya [1] investigated the characteristics of the wave propagation in periodic structures with imperfections.

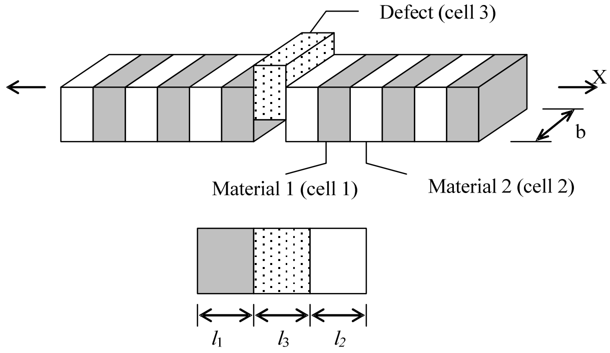
In this investigation, a combination of finite element modeling and the transfer matrix method is developed to solve the dynamics of the wave propagation of the periodic beam structures with defects. Hence, the finite element model is utilized to extract the transfer matrix that governs the propagation of waves along the periodic rod. The effects of various parameters on the pass and stop bands of a periodic structure with defects are calculated. The investigation in this study can provide the based guidelines to design the periodic structures with defects to obtain more effective filtering characteristics.

II. PROBLEM FORMULATION

As shown in Figure 1(a), a general periodic rod structure without defect is presented and the periodic rod structure with defect is shown in Figure 1(b). In this study, a finite element model is developed to describe the dynamic behaviors of the periodic system. Material 1 and material 2 of the periodic rod are assumed pure elastic and isotropic materials and another smart material - shape memory alloy (SMA) is also used in the composite rod structure and discussed in this study. In order to simplify the problem, the following assumptions must be mentioned first. The two parts of the periodic structures and the defect are subjected to longitudinal strain only.



(a) without defect



(b) with defect

Fig. 1. Geometry diagram of periodic rod

The dynamics of such one-dimensional periodic systems are determined using finite element modeling and transfer matrix method. The potential energy and kinetic energy of the cell 1, cell 2 and cell 3 are given as follows:

$$U_i = \frac{1}{2} b \int_0^{l_i} E_i h_i \left(\frac{\partial u_i}{\partial x} \right)^2 dx \text{ , } i=1, 2, 3 \tag{1}$$

$$T_i = \frac{1}{2} b \int_0^{l_i} (\rho_i h_i) \left(\frac{\partial u_i}{\partial t} \right)^2 dx \text{ , } i=1, 2, 3. \tag{2}$$

where u_i is the longitudinal displacement of the base rod of cell i.

In this study, a standard two nodes and two DOFs rod element has been used and the axial displacement u_i can be expressed in terms of nodal displacement vector $q(t)$, and shape function $N_i(x)$.

$$u_i = N_i(x) q(t) \text{ , } i=1, 2, 3 \tag{3}$$

where i denotes the number of the cells.

Then, the potential energy and kinetic energy of cell 1, cell 2 and cell 3 can be written as:

Cell *i*:

$$U_i = \frac{1}{2} \{q(t)\}^T [K_i] \{q(t)\} \text{ ,} \tag{4}$$

$$T_i = \frac{1}{2} \{\dot{q}(t)\}^T [M_i] \{\dot{q}(t)\} \text{ ,} \tag{5}$$

Where $[K_i] = \int_0^{l_i} b E_i h_i [\frac{\partial N_i}{\partial x}]^T [\frac{\partial N_i}{\partial x}] dx$,

$$[M_i] = \int_0^{l_i} b \rho_i h_i [N_i]^T [N_i] dx \text{ .}$$

In which $[K_i]$ and $[M_i]$ are the stiffness and kinetic matrices of cell 1, cell 2 and cell 3 respectively.

When the periodic section vibrates harmonically, with angular frequency ω, combining equations of cell 1, 2 and 3 yields the dynamic stiffness matrix of the stiffened rod element.

$$[K_{di}] = [K_i] - \omega^2 [M_i] \text{ ,} \qquad i=1, 2, 3. \tag{6}$$

The dynamics of wave propagation along the periodic rod structure with defect is determined by evaluating the propagation constants of the system, which are obtained by analyzing through the transfer matrix formulation. The dynamics of the elementary cells is represented in terms of the dynamic stiffness matrix as follows:

$$[K_{di}]\{Q_i\} = \{F_i\} \text{ ,} \qquad i=1, 2, 3 \tag{7}$$

and, the above relation can be rewritten as:

$$\begin{bmatrix} K_{diLL} & K_{diLR} \\ K_{diRL} & K_{diRR} \end{bmatrix} \begin{Bmatrix} Q_{iL} \\ Q_{iR} \end{Bmatrix} = \begin{Bmatrix} F_{iL} \\ F_{iR} \end{Bmatrix} \text{ , } i=1, 2, 3. \tag{8}$$

where K_{diLL}, K_{diLR}, K_{diRL} and K_{diRR} are the sub-matrices of K_{di}, respectively. F is the vector of generalized forces and Q is the vector of generalized displacement. The subscripts L and R denotes the left and right nodes of the cell i.

Then, Eqs. (8) can be rearranged to displacements and forces to yield the following expression for the transfer matrix of the periodic rod with defect and can be presented as follows:

$$Y_{kR} = T_k^m \cdot T_3 \cdot T_k^n \cdot Y_{kL} = T_s \cdot Y_{kL} \text{ ,} \tag{9}$$

where $Y_k = \begin{bmatrix} Q_i & F_i \end{bmatrix}^T$ denotes the state vector, and m, n represent the numbers of the entire cells of the periodic system. T_k is the kth entire cell transfer matrix (cell 1+cell 2), which is given by

$$T_k = T_k^{(1)} T_k^{(2)} \text{ ,} \tag{10}$$

Where

$$T_k^{(i)} = \begin{bmatrix} -K_{diLR}^{-1} K_{diLL} & K_{diLR}^{-1} \\ K_{diRR} K_{diLR}^{-1} K_{diLL} - K_{diRL} & -K_{diRR} K_{diLR}^{-1} \end{bmatrix} \text{, } i=1,2.$$

By the same way, the relation of the defect also can be obtained as follows

$$T_3 = \begin{bmatrix} -K_{d3LR}^{-1} K_{d3LL} & K_{d3LR}^{-1} \\ K_{d3RR} K_{d3LR}^{-1} K_{d3LL} - K_{d3RL} & -K_{d3RR} K_{d3LR}^{-1} \end{bmatrix} \tag{11}$$

Impose the following continuity and compatibility at the beginning of the entire cell,

$$\{Q_i\}_k = \{Q_i\}_{k-1} \text{ , } \{F_i\}_k = -\{F_i\}_{k-1} \text{ ,}$$

the relations of the entire cell can be expressed as follows:

$$Y_{(k-1)R} = J \cdot Y_{kL} \text{ ,} \tag{12}$$

where $J = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

Then, Eqs. (12) can be rewritten to obtain the relations at left and right of two consecutive entire cell:

$$Y_{kR} = J^{-1} \cdot T_s \cdot Y_{(k-1)R}\ ,= \overline{T}_s Y_{(k-1)R}\ . \qquad (13)$$

where $\overline{T}_s = J^{-1} \cdot T_s$. Afterwards, dropping the subscripts "*R*", Eqs. (13) can be written as a compact form:

$$Y_k = \overline{T}_s Y_{(k-1)}\ . \qquad (14)$$

III. RESULTS AND DISCUSSIONS

The composite rod of this investigation can be considered as a chain of cells according to the transfer matrix of the single cells. The transfer matrix $\overline{T}_s$ has two frequency-dependent eigenvalues, λ_1=e^{μ} and λ_2=$e^{-\mu}$, which determined the nature of the wave dynamics in the periodic rod with the defect.

In the above section, μ is the propagation constant of the periodic system. The real part of μ represents the decay of the amplitude of the wave propagation from one cell to the following cell and the imaginary part of μ determines the phase difference in two adjacent elements. The waves will propagate along the rod indicating the pass band and if not then the waves will be attenuated, indicating a stop band. The wave propagation of the periodic structures is possible within frequency band where μ has only imaginary part and attenuation occurs for the frequency band that provides a real part to the propagation constant.

To validate the proposed algorithm and calculations, comparisons between the present results and the results of existing models are made first. The numerical results are compared with those obtained by Ruzzene [9] in Table 1. The solutions solved by present model are shown to have a good accuracy.

In this study, the molded plastic is used to be as defects. The Young's modulus of the aluminum, steel, and molded plastic are *70GPa*, *210GPa*, *4GPa*, respectively. The densities of the aluminum, steel, and molded plastic are 2700kg/m^3, 7800kg/m^3, and 1726kg/m^3, respectively. The density of the SMA is 6500kg/m^3. The effect of the temperature on the Nitinol Young's modulus is shown in Figure 2, as measured experimentally inside a temperature-controlled chamber (Delta Design, Model 5900).

Table 1. Comparisons of the first stop band

	Boundaries of first stop band	Present (rad/s)	Reference [9] (rad/s)
25°C	Upper	620	~625
	Down	1365	~1360
50°C	Upper	620	~625
	Down	1410	~1400
75°C	Upper	620	~625
	Down	1520	~1530

In Figure 3, the stop bands of the periodic steel and SMA rod structure with defect are presented. Figure 3(a) and Figure 3(b) are the real and imaginary part of the propagation constant of the periodic system with defects. The physical and geometrical parameters are E_1=210*GPa*, E_2=30*GPa* (*25°C*), E_3=4*GPa*, l_2=5l_1=5*cm*, and h_1=h_2=h_3=1*cm*. The first stop band of the periodic system without and with defects is 110k-527k rad/s and 135-641k rad/s, respectively. But with defects, a small stop band will occur and the range of the additional stop bands is 97k-126k rad/s. It is an interesting point of the additional stop band as a result of the defects and we can use the characteristics to design a good acoustic wave filter.

The effects of the length ratios ($\lambda = l_3/l_1$, *where* l_1=1*cm*) on the propagation constants of the steel and SMA periodic system with a defect are plotted in Figure 4. The physical and geometrical parameters are E_1=210*GPa*, E_2=30*GPa* (*25°C*), E_3=4*GPa*, and l_2=5l_1. With different length of the defect, the location and width of stop band will be changed. The width of the stop band will be smaller with decreasing length of the defect and the stop band of the system will move forward with increasing length of the defect.

Finally, the effects of temperatures are discussed in Figure 5. The propagation constants of steel and SMA periodic rod with a defect at *25°C*, *50°C*, and *75°C* are plotted in Figure 5. The physical and geometrical parameters are E_1=210*GPa*, E_3=4*GPa*, l_2=5l_1=5*cm*, and h_1=h_2=h_3=1*cm*. As the temperatures increase, the width of first stop band of the system will decrease and move towards to left. The ranges of the stop band at *25°C*, *50°C*, and *75°C* are 135k-640k rad/s, 146k-642k rad/s, and 158k-644k rad/s.

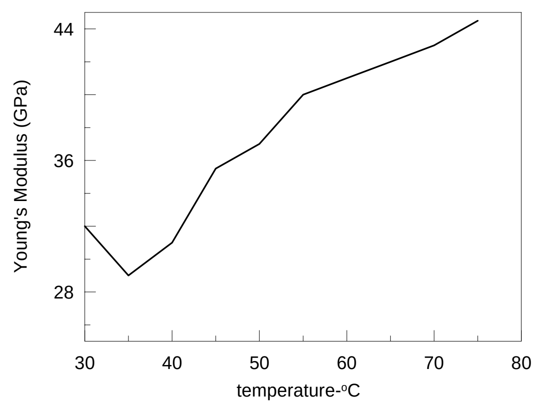


Fig. 2. Experimental evaluations of SMA Young's modulus against temperature [10]

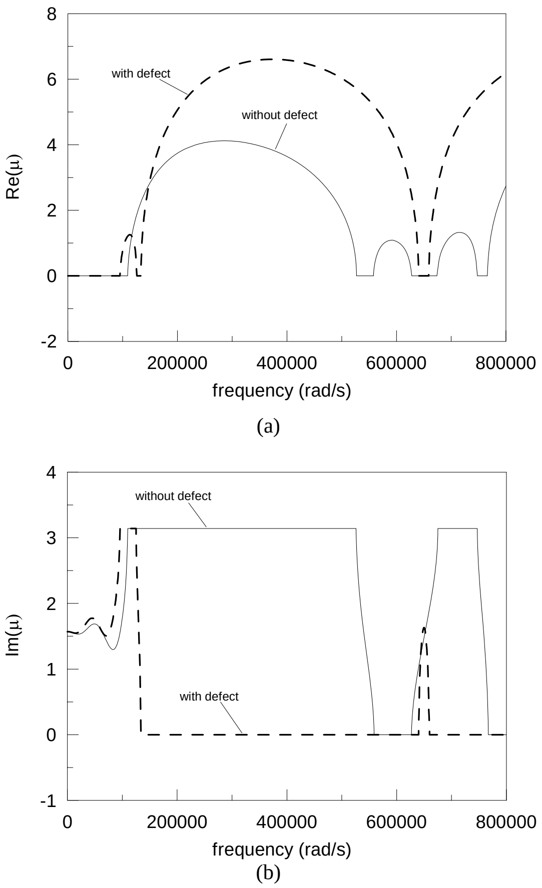


Fig. 3. Propagation constants of steel and SMA periodic rod system

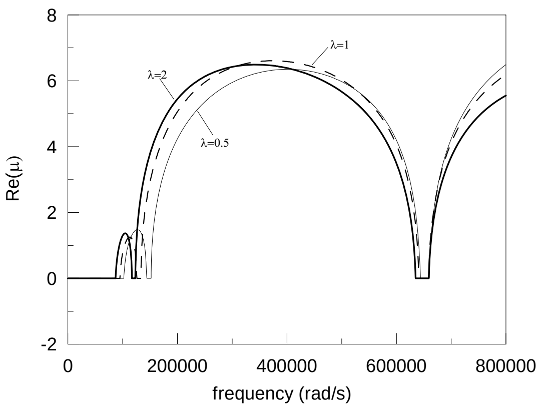


Fig. 4. Effects of length ratios λ on propagation constants of the steel and SMA periodic rod with defect

IV. CONCLUSIONS

The presented examples demonstrate the utility of the active and passive control capabilities in tuning the width and location of the pass and stop bands according to the nature of the external excitation. According to the results, it can be observed that we can filter the certain bands of frequencies by choosing a suitable periodic system. By adding a defect in the

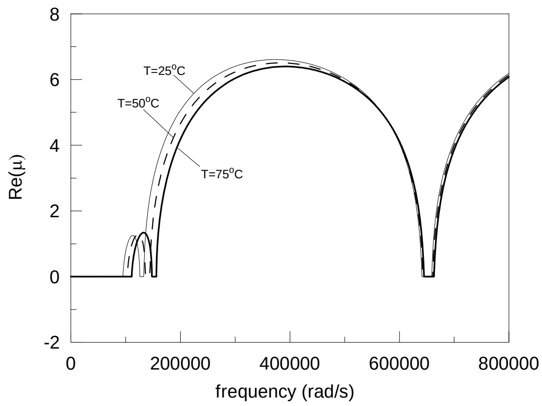


Fig. 5. Propagation constants of steel and SMA periodic rod with defect at various temperatures

periodic system, the stop band can be changed and filter different range of the bands of frequencies with various properties of the defects.

The techniques of the periodic sandwich structures have significant effects on the wave propagations and for controlling the flow of vibration and sound radiation to design a quiet and stable system. In present study, the investigation hopes to provide the basic guidelines to design periodic structures with various defects to achieve certain filtering characteristics.

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